SAR SIMULATION OF NATURAL LANDSCAPES

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ABSTRACT

This paper considers simulation of the SAR raw signal of natural landscapes. Within this framework, the format of the input data is critical. In fact, they are usually described by digital input maps which are too scarcely sampled for the application we are concerned. In our case, we deal with Digital Elevation Models, i.e., with the elevation input data. In order to interpolate such a data we employ the classical cubic spline interpolation and a fractal method, the latter based on the random midpoint displacement technique. Corresponding SAR images are generated and presented. Objective norms are further employed in order to compare simulated results with actual ones.

Keywords: Synthetic Aperture Radar (SAR), Simulation, Digital Elevation Model (DEM), Fractal geometry.

MOTIVATIONS

A SAR raw signal simulator is a powerful tool for both mission design and retrieval algorithms. To be useful, such a tool must rely on a sound electromagnetic model, which takes into account the surface-wave interaction, and on an efficient SAR system simulator [1].

Within this framework, SAR raw signal simulation of canonical scenes, i.e., described by sampled analytical functions [1], as well as of natural scenes [2], i.e., described by finite-resolution digital maps, are both of interest. In this paper we specifically address the second class of situations and the related problems.

A critical point is the accuracy of the input data involved in the simulation scheme. In particular, we focus on the Digital Elevation Model (DEM) input data. These are traditionally generated with an accuracy related to the original topographic map and usually too scarcely sampled for SAR simulation purposes [2]. In order to use such data we need to interpolate them [2]. Among several possible interpolation schemes, we employ an innovative fractal-based [3-6] one, suitable for natural landscapes [3-6], at variance with the conventional cubic spline method.

Simulation examples are presented and discussed. In particular, the accuracy of simulation is tested by comparison with actual images.

INTERPOLATION METHODS

In this Section we briefly outline the two interpolation methods we employed in order to generate the denser DEMs to be used as input elevation data to our SAR raw signal simulator [1].

The interpolation problem can be defined as the attempt to estimate the values of a function in between starting from its values at isolated points. In order to be attractive from the numerical point of view, the local interpolation schemes are of interest, i.e. schemes which requires knowledge of only a finite and possibly limited number of data.

Among several possible interpolation schemes, the polynomial interpolation is very often employed. This scheme assumes that the function to be interpolated belongs to the set of polynomia of finite degree q. In order to get a stable representation of the function to be interpolated, this is not usually described by a unique polynomium but the input data are subdivided in intervals and within each of them an interpolating polynomium is found.

A very popular polynomial interpolation is the spline one. Such an interpolation scheme estimates the polynomial coefficients by means of knowledge of the function and its p-derivatives at isolated points.

In particular, the cubic spline case is often encountered. In this case, q is equal to 3 while p is equal to 2. For the one dimensional case we use different polynomia in each sub-intervals $[x_{n-1},x_n]$ such that the resulting interpolating function results continuous up to its second derivatives included over the whole input data range $[a,b]$.

Extension to the two-dimensional case is straightforward and efficient algorithms can be found in order to implement such an interpolation method [7].

Former procedure is able to represent a deterministic function. In the case of natural surfaces random interpolation methods are more suitable. Specifically, it has been demonstrated that natural surfaces exhibit random fractalness and therefore a convenient method for their simulation is based on this property [3-6]. Hence, we consider a fractal-based interpolation method, in order to generate the denser DEM to be used in our simulator. This is the random midpoint displacement method that is a recursive method which allows to get a $2N \cdot 2M$ output matrix data from a $N \cdot M$ input digital map in a single step. We first have to locate the $N \cdot M$ input data point in a matrix of dimension $2N \cdot 2M$. The available data are located in the odd-odd matrix positions and we need to estimate the data in the even-even positions as well as in the odd (even)- even (odd) positions. Due to the different topological setup, two cases must be considered: in the even-even case, the data are found by means of the following formula:

$$z(n,m) = 1/4[z(n-1,m-1)+z(n+1,m-1)+ z(n-1,m+1)+z(n+1,m+1)]+\sqrt{1-2^{2H-2}}|\Delta x|^H \sigma\, G \quad , \tag{1}$$

while in the other case, the data are obtained by:

$$z(n,m) = 1/4[z(n,m-1) + z(n-1,m) + z(n+1,m) + z(n,m+1)] + 2^{-H/2}\sqrt{1-2^{2H-2}}\,|\Delta x|^{H}\,\sigma\; G \quad , \qquad (2)$$

wherein z is the elevation, and the matrix entries are the corresponding planar coordinates.

Eqs.(1) and (2) need some further clarification. The function G is a gaussian random variable with zero mean and unit variance, Δx is the two dimensional input data spacing, H is the Hurst exponent, which is related to the local fractal dimension and σ is a fractal Brownian model feature [6]. These latter quantities are locally estimated by means of a preliminary procedure whose rationale is given by the fractal Brownian model [3-6] which has been recognized to effectively represent natural surfaces [3-6].

THE SIMULATION ALGORITHM

In this Section we briefly outline the SAR raw signal simulator SARAS [1].

A sidelooking radar system scans the scene, characterized by its complex reflectivity map $\gamma(x,r)$[1], by periodically emitting modulated pulses and with a uniform[2] along track velocity. For a stationary (temporally invariant) reflectivity function, the raw signal $s(\cdot)$ can be written in the general form [1]:

$$s(x',r') = \iint g(x-x', r-r'; x,r)\; \gamma(x,r)dxdr \quad , \qquad (3)$$

where $g(\cdot)$ is the unit response function. Note that $g(\cdot)$ depends explicitly on the antenna pattern and on the impulse modulation [1].

A SARAS keypoint is the convenient evaluation of the reflectivity map; this is accomplished by means of an electromagnetic model rather than picking it up from a data base. This model is based on surface scattering computation and knowledge of surface topography and complex permittivity is required [1]. The surface relief is modelled by means of planar facets large in terms of the incident wavelength, so to correctly apply the Kirchhoff approximation for computing the backscattered field. The facets' density is properly chosen [1]. Hence, assuming an incident local plane wave over the single facet, we get [1]:

$$E_s = \frac{jk\exp(-jkR)}{4\pi R} E_o(\underline{\underline{I}} - \hat{k}\hat{k}) \cdot \int_A F(a,b,c)\exp[2jk\cdot\rho] , \quad (4)$$

where E_o is the incident field amplitude, k its vector wavenumber, $\underline{\underline{I}}$ the unit matrix, (a,b,c) the components of the normal to the facet A, and ρ its vector coordinate. The vector $F(\cdot)$ is a function depending on the Fresnel coefficients pertaining to the facet, hence it takes care of polarization issues: its expression is given in [1].

Efficient algorithms have been introduced to account for geometric distortion problems, without hampering processing time requirements [1].

An important point addressed in SARAS is the inclusion of statistical features. The echo radar from each facet, evaluated by means of eq.(4), must be considered as a realization of a stochastic process. First order and, if appropriate, higher order statistics must be taken into account, which correctly include roughness properties of the facet. An important case is the Rayleigh distributed scattered field, although other statistical classes may be considered. Inclusion of the statistical issues is very important, being related to the grainy appereance of the image (speckle). For a wider discussion on this issue, see Ref.[1].

Once the reflectivity map has been determined, computation of eq.(3) is in order. A two-dimensional approach is accomplished, in view of its ability to simply include SAR aberrations [1]. Numerical operations in the transformed Fourier Domain are convenient due to availability of powerful Fast Fourier Transform (FFT) codes. Eq.(3) becomes:

$$S(\xi,\eta) = G(\xi,\eta)\;\; \Gamma(\xi,\eta) \qquad (5)$$

where (ξ,η) are properly stretched [1] Fourier conjugate variables to (x,r) and capital letters indicate FTs of corresponding space domain functions. Stretching is necessary to take into account SAR aberrations [1].

The FT $G(\cdot)$ has been (asymptotically) evaluated in a closed form [1], and proper decompositions of FFT have been suggested [1] to cope with memory requirements and parallel processing. In this way, an efficient SAR raw signal simulator is possible, and an accurate simulation, including range migration, range curvature [1], and depth of focus effects [1] can be performed.

In summary, the simulation code consists of a first part, the most expensive from computation time point of view, which takes care of the evaluation of the reflectivity function. A second part performs the FT of $\gamma(\cdot)$ and mutiplies it by the (analytically evaluated) transfer function of the system, i.e., the FT of the unit response. The third final step computes the inverse FT to proceed to the output (x',r') domain.

EXAMPLES

The SAR simulator we formely described has been applied by using the ERS-1 mission data and the elevation data relevant to a mountainous area nearby Napoli, Italy. The input elevation data are sampled at 240 m · 240 m while the ERS-1 resolution is of the order of 4 m · 16 m. We generated two denser DEMs by means of the cubic spline interpolation method and the random midpoint displacement fractal-based method. Once these DEMs are obtained they can be inserted as input data to SARAS. Other scene input data are set as homogeneous and constant over the two cases. Corresponding SAR simulated images are depicted in Figs.1 and 2, respectively. In Fig.3 it is shown the corresponding actual image for comparison purposes.

As first comment we note that, in spite of the dramatically sparse input data, the results are remarkable. A visual test shows that the simulation based on the denser DEM achieved by means of the spline method provides good results, too. Comparative objective norms are advisable. To this end, we measured the ratio between the statistical mean and the standard deviation of four sub-regions (Tab.I). Region A is on the top left B on the top right, C on the bottom right, while D is on the bottom left of the images. Note that within each region a dominant mountain peak can be detected. In order to perform some geometrical norms (distances) over the three image planes we referred to these peaks; results are shown in Table II. In particular, the first two rows provide the distance (along azimuth) between the dominat peaks in regions A and

[1] This is the ratio between the backscattered and the incident field.

[2] Velocity perturbations may affect the flight, causing undesired changes, from pulse to pulse, in yaw, pitch and roll angles. In such case motion compensation techniques must be included in data processing.

B, and C and D, respectively. The other two rows refer to the distance (along range) between the dominant peaks in regions A and D and B and C, respectively. These results provide a remarkable and strong support to the use of the fractal-based interpolation method in order to achieve a denser DEM. Further systematic study is accomplished in Ref.[2].

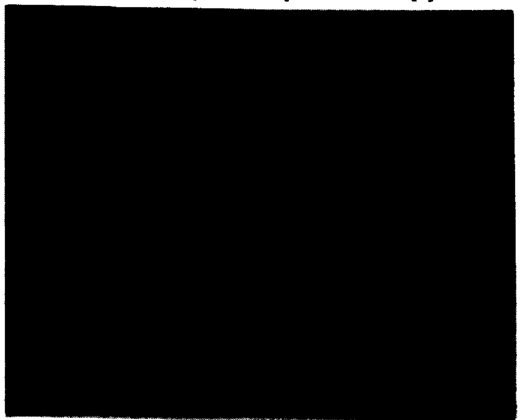

Fig.1: SAR simulated images whose DEM has been generated by means of the cubic spline interpolation method.

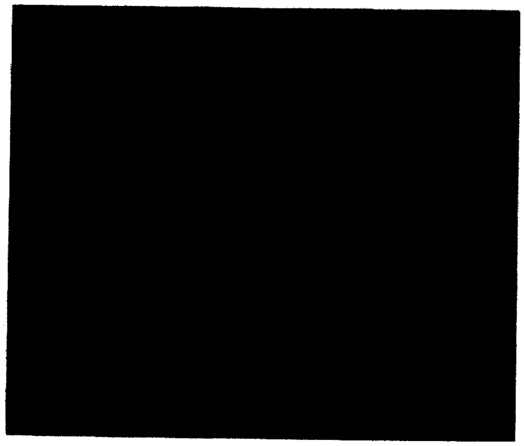

Fig.2: SAR simulated images whose DEM has been generated by means of the random midpoint displacement method.

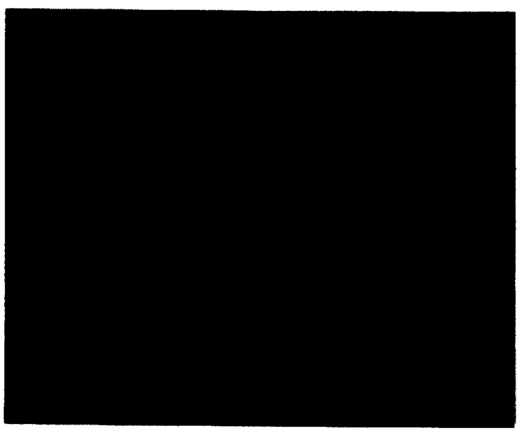

Fig.3: Actual SAR ERS-1 image.

	Actual	Fractal	Spline
A	2.878	2.220	1.491
B	2.046	1.778	1.585
C	2.505	2.173	1.354
D	2.597	2.398	1.279

Table I: Relevant to the ratio between the statistical mean and standard deviation over 4 sub-regions extracted from actual and simulated imagery.

	Actual	Fractal	Spline
A-B (azim.)	113	113	113
C-D (azim.)	122	122	122
A-C (range)	89	89	89
D-A (range)	74	74	74

Table II: Relevant to some geometrical distances (in pixels) measured over the actual and simulated SAR images.

SUMMARY

We considered in this paper the simulation of the SAR raw signal relevant to a natural landscapes. Within this framework the input data format is a relevant issue. In particular, we were concerned with the elevation data usually provided by means of a Digital Elelevation Model. These data are often too scarcely sampled for the application at hand; therefore appropriate interpolation methods must be employed. We tested the cubic spline and the random midpoint displacement methods. Objective norms over the simulated and actual images strongly support the use of the fractal-based interpolation method in order to get a denser DEM.

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