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A FRACTAL-BASED APPROACH TO ELECTROMAGNETIC MODELLING

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ABSTRACT

Mandelbrot introduced the notion of fractal dimension to describe, in a rigorous and quantitative way, the roughness of natural surfaces. Such a fundamental concept has suggested the application of fractal theory to characterize the scattering from rough surfaces. In this paper we deal with an appropriate electromagnetic scattering model suitable for microwave remote sensing applications.

Keywords: Electromagnetics, Scattering, Natural surfaces, Fractal geometry.

INTRODUCTION

In remote sensing community great relevance is given to the surface-electromagnetic wave interaction with natural surfaces [1]. Within this framework it is obviously unrealistic to assume that the surface morphology is limited by the scale of observation. To explore this intriguing problem some high-level stochastic approaches [2] turned out to be appropriate. However, some low-level sound physical approaches can now be envisaged. In particular, this is possible by means of the significant concept of fractal geometry which has been recently introduced by Mandelbrot [3]. As a matter of fact, it allows a viable and analytical direct insertion of the microscopic roughness within the scattering model [4].

The problem of plane wave backscattering from a natural terrain is investigated. We first model the macroscopic surface in terms of facets, large in terms of the incident wavelength, so to apply the Kirchhoff solution to the scattered far-field; then, we model the stochastic scattering contribution within each facet in terms of the Weierstrass-Mandelbrot fractal function [3].

Use of fractal concepts in electromagnetic scattering is not new [4]. However, for remote sensing applications, it is advisable to generalize the two-dimensional Weierstrass-Mandelbrot fractal function including random amplitude coefficients.

As a matter of fact, we note that in imaging radars a relevant point is the speckle effect. In order to take this into account, we need to properly model the fading occurring over the backscattered electromagnetic field. This is possible by means of a more complete stochastic characterization of the Weierstrass-Mandelbrot function, where amplitude and spatial frequency terms are not deterministic but random.

In addition, we note that natural surfaces are not well-modelled by perfectly conducting bodies, so that Fresnel reflection coefficients must be included in the scattering process.

The paper is structured as follows: first we outline the two-dimensional Weierstrass-Mandelbrot random fractal function. Then, we employ such a description in order to model the microstructure of the natural relief and to design a fractal-based approach to electromagnetic modelling. Subsequently, a number of examples are presented and discussed from qualitative and quantitative point of view. In particular, first-order statistical analysis is accomplished and compared with other available results. Finally, conclusions are collected.

THE SURFACE MODEL

In this Section we describe an appropriate fractal model for natural surfaces. It is based on the two-dimensional Weierstrass-Mandelbrot random fractal function [5,6], i.e.:

$$\zeta(x, y) = \sum_{i=1}^{\infty} C_i \lambda^{-\alpha i} \sin[\lambda^i (x \cos \Theta_i + y \sin \Theta_i) + \Psi_i] \quad (1)$$

wherein C_i , Θ_i , Ψ_i are three random variables which account for the random amplitude, the random frequency, and the random phase behaviours, respectively; λ (>1) is the seed of the geometrical progression which accounts for the spatial spectral components of the surface.

Some comments are in order. First of all, we stress that the Weierstrass-Mandelbrot random fractal function exhibits the self-similarity behaviour in the mean sense, that is:

$$\zeta(x, y) \approx \zeta(\lambda x, \lambda y) / (D - 2) \quad (2)$$

where,

$$D = 2 + \lambda^{-\alpha} \quad (3)$$

is the roughness fractal dimension of the surface.

In order to proceed further we need to specify the random coefficients. We reasonably assume Θ_i , Ψ_i to be uniformly distributed random variables over the whole trigonometric circle and C_i to be a zero-mean Gaussian random variable with unitary variance. Former random variables are assumed to be mutually independent. Accordingly, the $\zeta(x, y)$ variance is equal to:

$$\text{VAR}[\zeta(x, y)] = \frac{1}{2} \sum_{i=1}^{\infty} \lambda^{-2\alpha i} = \frac{1}{2(\lambda^{2\alpha} - 1)} \quad (4)$$

if $|\lambda^{-2\alpha}| < 1$. Obviously, to the purpose to model a surface with a standard deviation σ , the random amplitude coefficients must be spread according to the following standard deviation σ_c :

$$\sigma_c = \sigma \sqrt{2(\lambda^{2\alpha} - 1)} \quad (5)$$

Former relationship allows to link classical and fractal surface descriptor σ and D , respectively, and it is relevant on the applicative viewpoint.

In practical implementation of eq.(1), truncation to a finite number N of tones is necessary. This leads to a modified surface variance (see eq.(4)) and therefore eq.(5) must be recast, i.e.:

$$\sigma_c = \sigma \sqrt{\frac{2(\lambda^{2\alpha} - 1)}{1 - \lambda^{-2\alpha N}}} \quad (6)$$

We conclude that the Weierstrass-Mandelbrot random fractal function is a suitable candidates to model the rough microstructure in an efficient and sound physical manner.

THE ELECTROMAGNETIC MODEL

In this Section we consider the problem of evaluating the electromagnetic return from natural surfaces whose low spatial frequencies satisfies the conditions imposed by the Kirchhoff approximation and whose high spatial frequencies are described by means of the Weierstrass-Mandelbrot random fractal function.

Let the incident field be:

$$E_i = \hat{e} E_o \exp(-jk \cdot R) \quad (7)$$

wherein $k = k\hat{k}$ is the incident wave propagation vector, \hat{e} is a unit polarization vector, E_o is the incident field amplitude and R is the vector distance. The scattered field E_s is:

$$E_s = \iint_S \{ j\omega\mu_o \underline{\underline{G}} \cdot [\hat{n} \times H] + \nabla \times (\underline{\underline{G}} \cdot [\hat{n} \times E]) \} dS \quad (8)$$

where $\underline{\underline{G}}$ is the free-space dyadic Green function.

For microwave radar imaging, it is appropriate to use the natural facet model for the evaluation of the backscattered field, which, in the Kirchhoff physical optics approximation, is given by:

$$E_s = \frac{jk \exp(-jkR)}{4\pi R} E_o (I - \hat{k}\hat{k}) \cdot F(a, b, c) \int_A \exp(2jk \cdot \rho) dA \quad (9)$$

wherein (a, b, c) are the components of the normal to the facet A , and ρ the vector describing it. The function $F(\cdot)$ depends on the local Fresnel reflection coefficients as well as on the incidence angle, and polarimetry [7].

We have in general that the backscattering process over the facet introduces depolarization of the wave, so that we must generalize the reflectivity function in $(2 \cdot 2)$ matrix form:

$$\begin{bmatrix} E_s^H \\ E_s^V \end{bmatrix} = \gamma \begin{bmatrix} E_i^H \\ E_i^V \end{bmatrix} \quad (10)$$

with $E^{H,V}$ the horizontal and vertical polarized components of the electrical field. The reflectivity matrix can be written as follows [7]:

$$\underline{\underline{\gamma}} = D(\cdot) \cdot \underline{\underline{S}} \quad (11)$$

wherein $\underline{\underline{S}}$ takes into account the macroscopic polarimetric behaviour of the surface and $D(\cdot)$ describes the microscopic scattering behaviour of the elementary facet. For the $\underline{\underline{S}}$ entries we have [7]:

$$S_{HH} = \frac{2(a \sin \vartheta - b \cos \vartheta)}{a^2 + (b \sin \vartheta + c \cos \vartheta)^2} \cdot [a^2 R_q - (b \sin \vartheta + c \cos \vartheta)^2 R_p] \quad (12)$$

$$S_{HV} = S_{VH} = \frac{2a(c \sin \vartheta - b \cos \vartheta)(b \sin \vartheta + c \cos \vartheta)}{a^2 + (b \sin \vartheta + c \cos \vartheta)^2} \cdot (R_p + R_q) \quad (13)$$

$$S_{VV} = \frac{2(c \sin \vartheta - b \cos \vartheta)}{a^2 + (b \sin \vartheta + c \cos \vartheta)^2} \cdot [(b \sin \vartheta + c \cos \vartheta)^2 R_q - a^2 R_p] \quad (14)$$

wherein ϑ is the incidence angle, R_p and R_q are the Fresnel reflection coefficients related to the local incidence angle, the facet slopes and the electromagnetic permittivity, permeability and conductivity of the surface [7].

For the reirradiation diagram $D(\cdot)$ we have [7]:

$$D(\cdot) = \iint_A \exp(j2k \cdot \rho) dA \quad (15)$$

We now employ the formerly described Weierstrass-Mandelbrot random fractal function to describe the microstructure of the elementary facet of dimension $2X \cdot 2Y$ lying on the plane $z=0$. We have that this implies:

$$k \cdot \rho = (k_x x, k_y y, k_z \zeta) \quad (16)$$

and

$$D(\cdot) = \sum_{m_1, \dots, m_N = -\infty}^{+\infty} \exp(j \sum_{i=1}^N m_i \Psi_i) \prod_{i=1}^N J_{m_i}(u_i) \cdot 4XY \text{sinc}[(2k_x + \sum_{i=1}^N m_i \lambda^{-\alpha i} \Theta_i) X] \cdot \text{sinc}[(2k_y + \sum_{i=1}^N m_i \lambda^{-\alpha i} \Theta_i) Y] \quad (17)$$

wherein $u_i = 2k_z C_i \lambda^{-\alpha i}$.

ILLUSTRATIVE EXAMPLES

The first-order statistical behaviour of the fading associated to the backscattered electromagnetic field is investigated. The numerical study is based on the model sketched previously; we consider two similar cases which differ only for the fractal dimension. A facet size of $2m \cdot 2m$, and an electromagnetic plane wave, whose wavelength is 12.5 cm , impinging at 30°

degrees with respect to the normal to the mean plane have been considered. The surface microstructure has been described by 6 tones and a geometrical seed of $2e/3$ has been chosen. Fractal dimensions of 2.60 and 2.85 have been considered for the two cases at hand. In Figs.1(a) and 2(a) the surface microstructures are depicted, while in Figs. 1(b) and 2(b) the amplitude probability density functions (pdfs) of the backscattered electrical field are shown. In Figs.1(c) and 2(c) the phase pdfs are also shown.

As first result we note that both amplitude and phase pdfs are consistent with other available results [1,2], In particular, the amplitude pdfs remind the Rayleigh pdf, while the phase pdfs appear to be uniformly distributed. This suggests further systematic study in order to consider the fading dependence on the fractal dimension.

CONCLUSIONS

A fractal-based approach to electromagnetic modelling suitable for imaging radar applications has been depicted and some first illustrative examples have been presented. They suggest further systematic study in order to consider the fading statistical behaviour with respect to the fractal features.

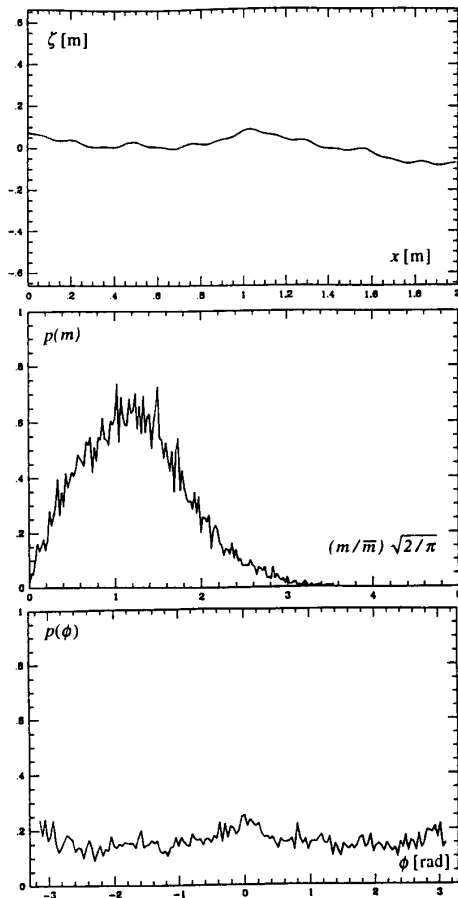


Fig.1:(a) Surface profile, (b) amplitude and (c) phase pdfs of the backscattered electrical field relevant to a microstructure of $D=2.60$.

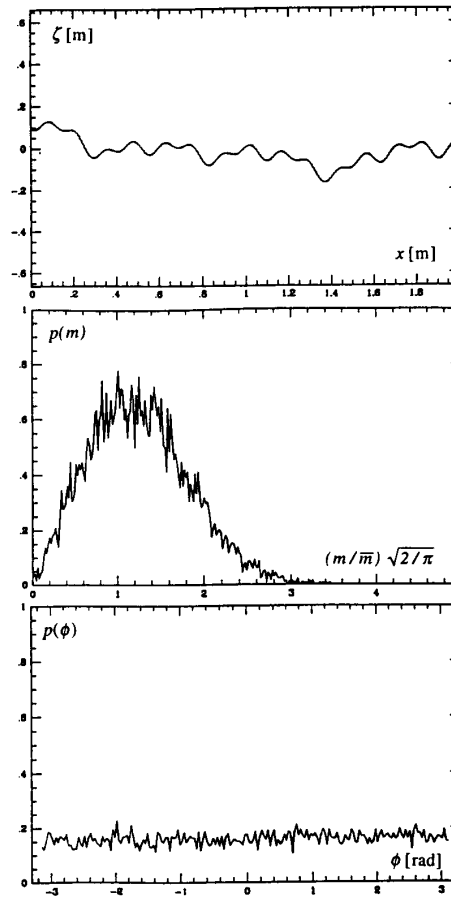


Fig.2:(a) Surface profile, (b) amplitude and (c) phase pdfs of the backscattered electrical field relevant to a microstructure of $D=2.85$.

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