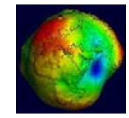
Image superresolution via filtered scales integral reconstruction applied to GOCE Geoid Data



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Abstract

In this work, a new approach is presented for enhancing enhancing image super-resolution via filtered scale integral reconstruction. Based on the concept that any sensed (from an instrumental point of view) specie must must be necessarily considered as an averaged value in a suitable time and space continuum, the local reconstruction approach expects to map a space of average values onto another one (filtered scales) with smaller extension and possibly, in an asymptotic limit, limit, onto the space of points. The only assumption is is that the measurements are given in the integral average sense i.e., for some reason, they have to be considered as integral average values for given computational domains or cells. The proposed algorithm can be used to correlate averaged values to point values; yet the most reliable results, in a probabilistic sense, can be obtained if the ratio between input and output scales is not too high. The essential concept of this working methodology is the possibility to correlate averaged measurements to averaged measurements upon suitably weighting the contribution of the molecules (or cells) surrounding the inspected one.

Mission description

The Gravity Field and Steady-state Ocean Circulation Explorer is an ESA mission whose primary objective is to measure the Earth's Geoid and gravity anomaly field with very high accuracy.
The satellite is planned for launch by the end of 2007, and will fly for approximately 1,5 years.
The satellite primary payload is composed by the Electrostatic Gravity Gradiometer (EGG), a three-arm gradiometer composed by highly sensitive accelerometers, and by a GPS receiver called called the Satellite-to-Satellite Tracking System (SSTI).GOCE will fly along a very low orbit, at a mean altitude of 250-240 Km and with inclination of 96.5°. Two main uses can be distinguished:

- The spatial variations of gravity and Geoid are directly related to density anomalies in lithosphere and upper mantle, respectively, and consequently to interior stresses and ultimately to mass motion. In this respect GOCE provides important new information information to studies of continental and oceanic lithosphere and upper mantle. Its information is complementary to that of seismic tomography, magnetic field models, geo-kinematic studies and laboratory results.
- Secondly, a detailed Geoid surface when combined with satellite altimetry yields sea surface topography, the quasi-stationary deviation of the ocean surface from its hypothetical surface of rest. Under the assumption of geo-strophic balance ocean topography can be directly translated into a global map of surface ocean circulation. Thus, ocean surface circulation becomes directly measurable, globally and uninterruptedly. In conjunction with higher resolution ocean models and ocean measurements, GOCE is expected to improve significantly estimates of global mass and heat transport in the oceans.

The Gravity Field Model

The GOCE gravity field models are provided in terms of a set of dimensionless coefficients of a spherical harmonic harmonic series up to a maximum degree of the gravity potential. These coefficients are the result of the gravity field determination process. All other quantities quantities delivered together with the products are derived from these coefficients.

- The coefficients of the spherical harmonic series of the gravitational potential are provided as (see Rummel et al., 2002)

$$V(r,\theta,\lambda)=\frac{GM}{r}\sum_{n=0}^{N\max}\left(\frac{a}{r}\right)^{n}\sum_{m=0}^{n}\left(\overline{C}_{nm}\cos m\lambda+\overline{S}_{nm}\sin m\lambda\right)\overline{P}_{nm}(\cos\theta)$$

- Geoid Heights are defined as (see Rummel et al., 2002):

$$N(r,\theta,\lambda)=\frac{GM^{REF}}{r\gamma_0}\sum_{n=0}^{N}\left(\frac{a^{REF}}{r}\right)^{n}\sum_{m=0}^{n}\left(\overline{\Delta C}_{nm}\cos m\lambda+\overline{\Delta S}_{nm}\sin m\lambda\right)\overline{P}_{nm}(\cos\theta)$$

- The Spatial Resolution is linked to maximum resolvable degree "l" as follows (see Rummel et al., 2004) :

$$D=20000/l$$

Earth Geoid from GOCE

The European Space Agency has defined a ground system, which will prepare and process the GOCE observations up up to so-called level 2 products. Main Level 2 products products will be:

- Precise Science Orbits
- Gravity Gradients in different reference frames
- Gravity field models as well as quality indicators associated to them.

The GOCE L2 product containing the Geoid Model is called EGM_GOC_2_ . The gravity field is provided as spherical harmonic series up to a specific degree and order. Derived quantities are provided on equi-angular grids. The quality report is provided in PDF format.

The product content is:

- Spherical harmonic series including error estimates
- Grids of Geoid heights, gravity anomalies and deflections of the vertical
- Propagated error estimates in terms of Geoid heights
- Quality report for GOCE gravity field model

Integral Reconstruction Algorithm Description

Integral reconstruction algorithm is based on the hypothesis that any signal is a mean value relative to a backscattering cell as in formula here-after. Every cell is part of a macromolecule centered in the cell itself which contributes numerically to the local integral bidimensional regressive strategy:

$$\varphi_i=\frac{1}{\mu(\Omega_i)}\int_{\Omega_i}\varphi(P)d\omega$$

where $\varphi_i\in F_{i-k,i-k+1\ldots,i,i+j-1,i+j}$ is a function definite in the macromolecule centered in i and possibly spreading around it it in a concentric fashion. F is the approximant family (polynomial, fourierian, etc…) which is defined with n degrees of freedom depending on its order, i.e. the local expansion order. It represents an expansion in a suitable suitable basis:

$$\varphi_i=\sum_{i=1}^{s}\lambda_i(\Omega_{i-k},\Omega_{i-k+1},\ldots,\Omega_{i+j})\,\overline{\varphi}_i$$

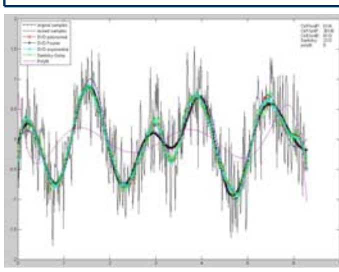
where λi depends on all cells in the macromolecule; its value is obtained with a LS (Least Squares) approach via an an SVD decomposition which minimizes, in the euclideian norm, the reconstruction error:

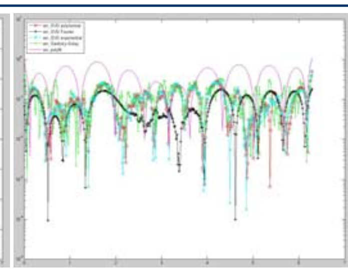
$$\left\|\underline{\underline{C}}\,\underline{\lambda}-\underline{b}\right\|_2=\left\|\underline{\underline{U}}^T\left(\underline{\underline{A}}\,\underline{\underline{V}}\,\underline{\underline{V}}^T\underline{\lambda}-\underline{b}\right)\right\|_2$$

After that a possible integral 2D-regression can be performed:

- in a mean value-to-point way: $\varphi_{\Omega_i}\rightarrow\varphi(P)$ yielding a function of a point, in output, from a series of mean integral integral values, in input.
- in a mean value-mean value way: $\varphi_{\Omega_i}\rightarrow\varphi_{\Lambda^j}$ where $\Lambda_i^j\in\{\Lambda\}_j$ is a measure decreasing set of domains contained in λi defining the so called "filtered scale" approach.

The following figures depict a typical reconstruction result obtained with a set of 1D values; these ones originates e.g. from a typical DEM survey. One can see the confrontation of a series of different algorithms based, fist of all, on the two main "philosophical" approaches: simple interpolation and integral reconstruction.
A multiple reconstruction families strategy has been adopted in order to test several regressive bases. Depending on the particular problem an integral reconstruction family can yield the best values with respect to the remaining remaining ones. Yet what generally happens is that the reconstruction error, by using a known test signal, is the lowest with respect to classical interpolation (see figures).



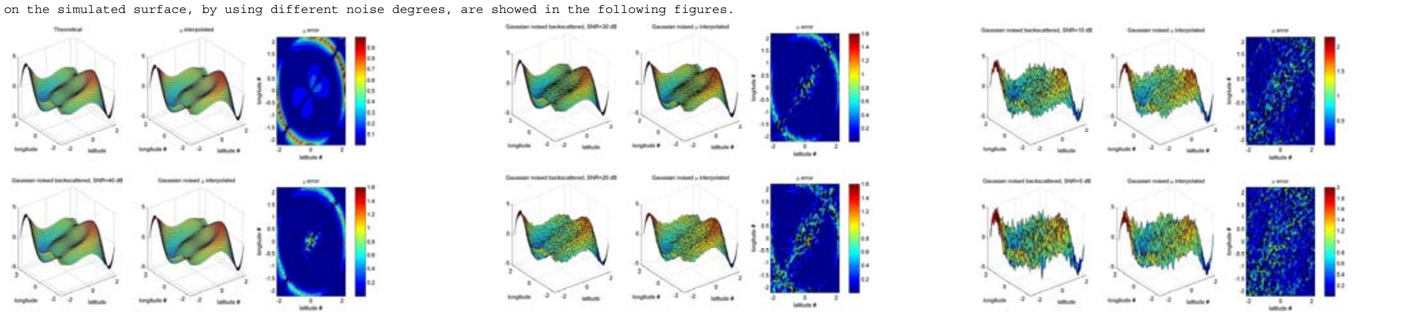


Geoid Product Specification:

- Reference System: WGS84 reference ellipsoid for Geoid heights, gravity anomalies and deflections of the vertical.
- Reference Frame: EFRF
- Spatial coverage: Spherical harmonic series: global per definition. Grids: limited by GOCE inclination (±82° latitude)
- Spatial resolution: Derived quantities will be provided as 30'x30' equi-angular grid
- Latency: 9 months after completion of each measurement operational phase

Geoid reconstruction

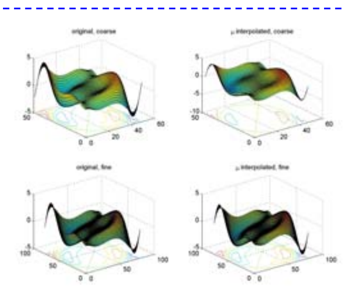
The procedures described above has been applied also to an analytical function representing a portion of the Earth Geoid surface. The results of application of the integral reconstruction method on the simulated surface, by using different noise degrees, are showed in the following figures.



The Figures above depict a typical μ-regressive application. On the left columns one can see a test two-dimensional profile representing a peak area, in the central column the relative μ-regressed samples series whilst in the right column the relative error with respect to the max abs value of the current working macromolecule.

The Figures here-after show meshes with different granularities and corresponding reconstructions.

- On the **left column:** two meshes relative to the same inspected area but with different granularity (under-sampled respect to the surface showed in above figures) figures) have been depicted ("original" stands for analytical). On the **right column** the integral regressive reconstruction algorithm has been applied for each of the base lattices, respectively.
- It is important to highlight that both of the μ-interpolated lattices well approximate the surface in the inner region whilst, as concerns peripheral areas, only the refined μ-lattice is correct. That is not absolutely a problem since the μ-μ-regressive algorithm is typically related and used for inner cells. In other words words it is possible to span the entire inspected areas by resorting to a multiple application of the μ-regression on several sub-domains. Recall that the "μ-interpolated-fine" gridding has been obtained directly from "μ-interpolated-coarse" considering that in each cell the behavior of the shape is completely know since the the function is analytically locally defined.
- This exactly explains what "filtered scale" can achieve : from a mean value defined, as an integral value, in a wider cell you can obtain another integral mean value defined in a smaller cell.



Conclusions:

- The Integral Reconstruction Algorithm performance and characteristics are suitable for Reconstructed Geoid Computation
- The Errors obtained in applying proposed algorithm on analytical surface is limited to acceptable range. The same algorithm performances are foreseen by using GOCE Geoid Data
- The application of the proposed algorithm on GOCE Geoid data is aimed at enhancing the spatial resolution of the Geoid field
- Future work will be devoted to customize the algorithm in order to "process" real GOCE Data (non-analytic surface)

References and acknowledgements:

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