A Waveform Model for Near-Nadir Radar Altimetry Applied to the Cassini Mission to Titan

Giovanni Alberti, Luca Festa, Claudio Papa, and Guido Vingione

***Abstract*—The radar altimeter of the Cassini mission to Titan operates in a transition region between pulse- and beam-limited conditions. Due to the specific observation geometry, low values of mispointing angle have been found to significantly affect altimeter impulse response (IR). This involves a nonconventional formulation of the system response which is the main goal of this paper. An analytical model of the average return power waveform, valid for near-nadir altimetry measurements, has been developed in order to cope with the particular operating conditions of Cassini mission. The model used to approximate the altimeter waveform is based on the same general assumptions of the classical Brown's model (1977) but exploits a flat surface response approximation by Prony's methods. Both theoretical considerations and simulated data have been taken into account to support the accuracy of the proposed model. To infer the main geophysical parameters describing surface topography from altimetry data, a parametric estimation procedure has been used. The maximum likelihood estimator procedure has been chosen since, in principle, it can assure optimal performance as a consequence of the analytical model we used to describe the system IR. Performances of the implemented method have been numerically evaluated through simulation of data received by CASSINI in high-resolution altimeter mode.**

***Index Terms*—Maximum likelihood estimation (MLE), planets, radar altimetry, radar data processing, remote sensing, spaceborne radar.**

I. Introduction

THE CASSINI radar [1], [2] is a multimode instrument designed to investigate the inaccessible surface of Titan, Saturn's largest moon. The instrument operates onboard the Cassini–Huygens mission, an international project involving the National Aeronautics and Space Administration (NASA), the European Space Agency (ESA), and the Agenzia Spaziale Italiana (ASI). The altimeter mode aims to study the relative topographic change of Titan's surface along subsatellite tracks.

Before the Cassini mission, spaceborne radar altimeters have been commonly used on Earth to map Earth's geoid, to study oceanic processes, to obtain topographic details of ice, land, and sea surfaces, and to monitor and collect data concerning various global processes [10].

It is well known that the characteristics of an altimeter waveform are strongly related to surface statistical properties (i.e., roughness, rms slope, etc.). In principle, this means that the information content carried by a received echo can be extracted if we make use of a model of the altimeter's echo waveform.

In 1977, Brown proposed a theoretical model of the average impulse response (IR) of a rough surface, the so-called Brown's model, which has been widely applied to pulse-limited radar altimeters devoted to nadir ocean observations [8].

Due to mission constraints, the Cassini radar altimeter works so that pulsewidth-limited and beam-limited circles are comparable [6], [13]. In this situation, some general assumptions of conventional models are not applicable for Cassini radar. Furthermore, the geometry involved, mainly the effective attitude of the Cassini orbiter during the hyperbolic Titan fly-bys and the nonnegligible off-pointing angles, strongly affects the waveform shape and, hence, the final altimetric measurements. This implies the demand for a surface IR approximation of [7], which incorporates all those effects and admits a straightforward closed-form solution.

For off-nadir measurements, Brown [7] showed the flat surface IR (FSIR) approximation by means of an asymptotic expression obtained by means of the Laplace's method, with an error lower than 2% of true value when dealing with far off-nadir pointing angles [16].

Using the same hypotheses made by Brown in [8], but with a different approach, Montefredini *et al.* [13] made a model based on a series expansion of the Bessel function, not dependent on the radar operating condition and also suitable in case of large mispointing. However, the final numerical expression makes the implementation of any parametric estimation procedures difficult.

The surface Titan profile obtained by processing Cassini altimetric data has also been analyzed showing its fractal behavior [3].

Following a short description of the Cassini altimetric mission, a new closed-form solution for altimeter waveform to be used in case of near-nadir measurements is presented. A comparison with respect to the ocean-type Brown's model is also showed. A model-related error budget has been assessed with respect to numerical solution. The developed analytical model is exploited to estimate surface height and sigma nought by means of a maximum likelihood (ML) method. The implemented algorithm is described, and its performance is evaluated by means of simulated echoes of the Cassini radar in altimeter mode.

II. Cassini Mission to Titan

Titan is the only satellite in the solar system with an appreciable atmosphere, composed mostly of nitrogen, aerosols, and a variety of hydrocarbons. Its surface is believed to feature

Manuscript received January 30, 2008; revised May 28, 2008, September 11, 2008, and November 25, 2008. First published April 28, 2009; current version published June 19, 2009. This work was supported in part by the Italian Space Agency (ASI).

The authors are with the Consortium for Research on Advanced Remote Sensing Systems (CO.RI.S.T.A.), 80125 Naples, Italy (e-mail: alberti@unina.it).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TGRS.2009.2012718

TABLE I
MAIN PARAMETERS FOR THE HIGH-RESOLUTION CASSINI ALTIMETER

Frequency	13.78 GHz
Antenna beamwidth (θ_{3dB})	0.350 deg (6.1 mrad)
Sampling frequency (f_c)	10 MHz
Chirp length (T)	150 µs
Chirp bandwidth (B)	4.25 MHz
Range (vertical) resolution (ρ)	35.3 m

chilled lakes of mainly methane, with a small amount of ethane, and a surface coated with sticky brown organic condensate that has rained down from the atmosphere [12]. Due to a dense hydrocarbon haze that forms in the stratosphere as methane is destroyed by sunlight, Titan's surface has been very difficult to study until now.

The Cassini radar is designed to operate in four observational modes (imaging mode, altimeter mode, scatterometer mode, and radiometer mode) at various spacecraft altitudes on both inbound and outbound tracks of each hyperbolic targeted Titan fly-by, in order to accommodate the potentially different types of surface [2]. Operating at spacecraft altitudes between 4000 and 9000 km, the altimeter mode utilizes the central nadir pointing antenna beam for transmission and reception of chirp burst pulse signals.

Instrument nominal main parameters used for the purpose of the present work are summarized in Table I.

If we consider the nominal operating altitudes of the Cassini orbiter, we find that the radii of the pulsewidth-limited and of the beamwidth-limited circle are comparable, according to the study in [13]. For instance, considering a nominal altitude of 6000 km, we obtain, in two cases, a footprint diameter of about 41.2 and 36.5 km, respectively.

III. WAVEFORM MODEL DEVELOPMENT

The general assumptions at the basis of the development of the altimeter echo model hereafter described are as follows [8]:

1) completely noncoherent nature of the scattering mechanism [15];
2) independent scattering elements on the observed surface;
3) rough surface with Gaussian height probability density function;
4) backscattering cross section per unit scattering area (σ^0), depending only on incidence angle;
5) negligibility of Doppler frequency spreads;
6) antenna beam circularly symmetric with Gaussian antenna gain pattern with respect to off-nadir angle θ, i.e.,

$$G(\theta) \approx G_0 \exp\left(-\frac{2}{\gamma}\sin^2\theta\right) \tag{1}$$

$$\gamma = -\frac{2\sin^2(\theta_{3\,\mathrm{dB}}/2)}{\ln(0.5)} \tag{2}$$

where G_0 is the peak antenna gain (at boresight) and $\theta_{3\,\mathrm{dB}}$ is the -3-dB antenna aperture.

In order to obtain the average altimeter echo, for both nadir and off-nadir pointing observations, the convolution of the following three terms must be evaluated (*convolutional model* [8], [14]):

1) the FSIR;
2) the radar point target response;
3) the probability density function of the height of the specular points on the observed rough surface.

The expression of the FSIR, including the radar mispointing (ξ), is given by the study in [8] as a function of the two-way incremental ranging time $\tau = t - 2h/c$

$$\begin{cases} P_{\mathrm{FS}}(\tau) = K_{\mathrm{FS}} \exp\left(-\frac{4c\tau}{\gamma h\Lambda}\cos 2\xi\right) & \\ \quad\cdot I_0\left(\frac{4}{\gamma}\sin 2\xi\sqrt{\frac{c\tau}{h\Lambda}}\right), & \tau \geq 0 \\ P_{\mathrm{FS}}(\tau) = 0, & \tau < 0 \end{cases} \tag{3}$$

where

$$K_{\mathrm{FS}} = \frac{G_0^2\lambda^2 c\sigma^0(\psi_0)}{4(4\pi)^2 L_p h^3}\exp\left(-\frac{4}{\gamma}\sin^2\xi\right) \tag{4}$$

$$\psi_0 \approx \tan^{-1}(\sqrt{c\tau/h}). \tag{5}$$

Here, c is the speed of light, λ is the radar carrier wavelength, L_p is the two-way path loss, h is the satellite altitude above the mean flat surface, and σ^0 is only dependent on the observation angle ψ_0 that can be neglected in case of small observation angles (i.e., σ^0 is constant over the effective illuminated area). As far as the last assumption is concerned, it is worth noting that, even the incidence angle is affected by local terrain slope and satellite attitude, models for electromagnetic scattering from natural rough surfaces show negligible variation of σ^0 (tenths of decibels) up to few degrees [5]. The last expression is valid for

$$\sqrt{\frac{c\tau}{h}}\tan\xi \ll 1 \tag{6}$$

that is well verified in the case of Cassini fly-bys. The only difference with respect to the classical formulation of Brown is the inclusion of the spherical surface effects that implies [16] a formal substitution of τ with τ/Λ, being

$$\Lambda = (1 + h/R_T) \tag{7}$$

where R_T is the mean radius of Titan (2575 km).

Provided that P_T is the peak transmitted power and B and T are the transmitted bandwidth and pulsewidth, respectively, the system IR can be evaluated by taking the convolution of the FSIR with the convolution P_{HI} between the height probability density function and the system point target response, both supposed to be Gaussian

$$P_{\mathrm{HI}}(\tau) = K_{\mathrm{HI}}\exp\left[-\frac{(\tau-\tau_0)^2}{2\sigma_c^2}\right] \tag{8}$$

where

$$K_{\mathrm{HI}} = P_T BT\sqrt{2\pi}\frac{\sigma_p}{\sigma_c} \tag{9}$$

$$\sigma_c = \sqrt{\frac{4}{c^2}\sigma_h^2 + \frac{1}{8\ln 2B^2}} \tag{10}$$

with the parameter σ_c related to either the rms height of the specular points relative to the mean reference surface (σ_h) and to system vertical resolution ($1/2B$).

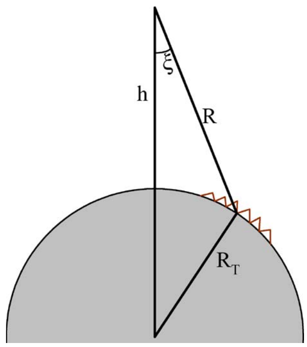

Fig. 1. Off-nadir altimetry geometry.

In the last expression, the extra delay τ_0 takes into account the time shift of height given, as shown in Fig. 1, by

$$\tau_0 \approx \frac{2}{c}(h + R_T)(\cos\xi - 1) + \frac{(h + R_T)^2 \sin^2\xi}{cR_T}. \tag{11}$$

Of course, the extra delay τ_0 vanishes for nadir pointing altimetry, i.e., for $\xi = 0$. In the following, all the evaluated IR functions have to be considered delayed by the previous extra amount of time.

The convolution integral cannot be solved analytically in the general case, except if a nadir pointing configuration is considered $\xi = 0$. In this case, after some manipulation [4], the IR becomes

$$IR^{\mathrm{Nadir}}(\tau) = P_{\mathrm{FS}}(\tau)\,|_{\xi=0} * P_{\mathrm{HI}}(\tau) \tag{12}$$

$$IR^{\mathrm{Nadir}}(\tau) = K\sigma^0 \frac{1}{2}\exp\left(-\frac{\delta}{\sigma_c}\tau + \frac{\delta^2}{2}\right) \cdot \left[1 + \mathrm{erf}\left(\frac{\tau}{\sqrt{2}\sigma_c} - \frac{\delta}{\sqrt{2}}\right)\right] \tag{13}$$

where

$$K = \frac{G_0^2 \lambda^2 c P_T T}{32\pi\sqrt{8\ln 2}\, L_p h^3} \tag{14}$$

$$\delta = \frac{4c}{\gamma h \Lambda}\sigma_c. \tag{15}$$

The aforementioned equation can be considered as a generalization of the classical Brown's model [8]. In fact, it corresponds exactly to the Brown's solution if the argument of the erf function can be simplified, i.e.,

$$\frac{\tau}{\sigma_c} \gg \delta. \tag{16}$$

The last expression, if verified for the minimum time delay ($\tau_{\min} = 1/B$) and for flat surface ($\sigma_s = 0$), can be rewritten as

$$\frac{4c}{\theta_{3\,\mathrm{dB}}^2 Bh} \ll 1 \tag{17}$$

which is the pulse-limited condition.

In case of an off-nadir pointing radar altimeter ($\xi \neq 0$), the FSIR evaluation cannot be simplified. The most practical method of evaluation of the average return power waveform should be numerical integration of the general expression given by Newkirk and Brown [16].

When dealing with far off-nadir pointing angles, a closed asymptotic form for the FSIR can be derived [7] by using Laplace's method [17]

$$P_{\mathrm{FS}}^{\mathrm{Asymp}}(\tau) = \frac{G_0^2 \lambda^2 c\sigma^0(\psi_{\cdot 0})}{2(4\pi)^3 L_p h^3} G(\varepsilon), \quad \tau \geq 0 \tag{18}$$

where

$$G(\varepsilon) = \exp\left(-\frac{4\left(\sin\xi - \varepsilon\cos\xi\right)^2}{\gamma(1+\varepsilon^2)}\right)\sqrt{\frac{2\pi}{a+2b}} \tag{19}$$

$$a = \frac{4\varepsilon}{\gamma}\frac{\sin 2\xi}{(1+\varepsilon^2)} \quad b = \frac{4\varepsilon^2}{\gamma}\frac{\sin^2\xi}{(1+\varepsilon^2)} \quad \varepsilon = \sqrt{\frac{c\tau}{h\Lambda}}. \tag{20}$$

For high values of off-nadir angle, the asymptotic expression of FSIR waveform becomes much wider than both the surface height distribution and the pulse response. Therefore, the total IR can be simply written as the following product:

$$IR^{\mathrm{Asym}}(\tau) = \frac{K\sigma^0 G(\varepsilon)}{2}\left[1 + \mathrm{erf}\left(\frac{\tau}{\sqrt{2}\sigma_c}\right)\right], \qquad \tau \geq 0. \tag{21}$$

The problem of finding a closed form for the IR still exists for small mispointing angles. A possibility, also suggested by Brown [8], consists in approximating the FSIR by a series of exponentials by using the classical Prony's method [11].

That approach is followed in this paper, where only the Bessel function has been approximated by using Prony's method. In fact, by transforming the Bessel function of the FSIR to an appropriate exponential function allows one to close the convolution integral.

The starting point is the FSIR expression of (3), which can be rewritten as a function of the nondimensional parameter ε

$$P_{\mathrm{FS}}(\varepsilon) = K_{\mathrm{FS}} \exp\left(-\frac{4}{\gamma}\varepsilon^2\cos 2\xi\right) \cdot I_0\left(\frac{4}{\gamma}\varepsilon\sin 2\xi\right), \qquad \varepsilon \geq 0. \tag{22}$$

After some tradeoff, it has been found that the most convenient way of approximating the Bessel function is the following:

$$I_0\left(\frac{4}{\gamma}\varepsilon\sin 2\xi\right) \approx \sum_{i=1}^{N} C_i \exp[a_i x] \tag{23}$$

$$x = 4\sin(2\xi)\varepsilon^2/\gamma \tag{24}$$

where the constants C_i and a_i are evaluated with Prony's method.

In this way, the FSIR can be written as a simple summation of N exponential terms, such as

$$P_{\mathrm{FS}}^{\mathrm{Prony}}(\varepsilon) = K_{\mathrm{FS}}\exp(-K_a\tau)\sum_{i=1}^{N} C_i \exp(K_i\tau) \tag{25}$$

where

$$\begin{cases} K_a = \frac{4}{\gamma}\cos(2\xi)\frac{c}{h\Lambda} \\ K_i = \frac{4}{\gamma}\sin(2\xi)\frac{c}{h\Lambda}a_i. \end{cases} \tag{26}$$

For example, with reference to the main system parameter of Table I and by considering an altitude of the spacecraft of

TABLE II
EXPONENTIAL AND AMPLITUDE FACTORS FOR PRONY'S APPROXIMATION

	N=2	N=3	N=4	N=5
C_i	0.47-3.56j 0.47+3.56j	9.10 -4.05-0.91j -4.05+0.91j	1.30-19.73j 1.30+19.73j -0.80+4.90j -0.80-4.90j	54.52 -31.67-2.47j -31.67+2.47j 4.91+0.32j 4.91-0.32j
a_i	50.01+22.69j 50.01-22.69j	43.98 31.58+27.76j 31.58-27.76j	33.35+9.73j 33.36-9.733j 19.86+24.94j 19.86-24.94j	29.65 25.32+13.55j 25.32-13.55j 13.18+21.94j 13.18-21.94j

5000 km and an off-nadir angle of 0.15°, Table II shows the amplitude and exponential factors computed by the Prony's approximation of all orders. It is worth noting that, in general, these factors can be complex but conjugated in pairs. Therefore, the FSIR is a combination of exponential and sinusoidal terms given by the real and imaginary parts of K_i, respectively. As expected, the overall summation of the N exponential terms gives real results.

Now, the convolution integral can be evaluated as done for (13). In an analogous way, the following parameter can be defined:

$$\delta_i = (K_a - K_i)\sigma_c \tag{27}$$

and the final IR can be written as

$$IR^{\text{Prony}}(\tau)=\frac{K\sigma^0}{2}\exp\left(-\frac{4}{\gamma}\sin^2\xi\right) \cdot\sum_{i=1}^{N}C_i\exp\left(-\frac{\delta_i}{\sigma_c}\tau+\frac{\delta_i^2}{2}\right)\left[1+\text{erf}\left(\frac{\tau}{\sqrt{2}\sigma_c}-\frac{\delta_i}{\sqrt{2}}\right)\right]. \tag{28}$$

Following what was said before, the erf function of (28) should be extended to complex argument [4].

The aforementioned analytical equation allows for approximating the IR in case of near off-nadir pointing angles. In the case of the Cassini radar, negligible approximation errors are reached with few terms (four at the maximum), as shown in the following section. This allows an easy and rapid calculation of either the impulse function or its derivative needed for the estimation procedure.

It is worth noting that, for all cases corresponding to (13), (21), and (28), the averaged IR can be written in the following way, by underlining the dependence of main surface parameters:

$$IR^M(\tau) = A(\sigma^0)f_M(t_0,\xi,\sigma_s). \tag{29}$$

Some examples of such calculations and a full assessment of errors involved by the previous models are contained in the next paragraph.

IV. IR Calculation and Related Error Budget

In the previous paragraph, three different models in analytical closed form have been formulated, corresponding to (13), (21), and (28). Only the last two models have been developed originally by the authors of this paper.

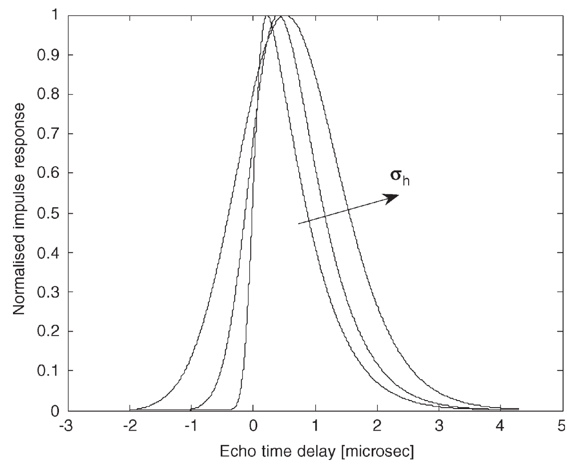

Fig. 2. Theoretical IR ($H = 5000$ km and $\xi = 0.15°$) for various surface height rms values ($\sigma_h = 10$ m, 50m, and 100m).

The aim of the present paragraph is to show some examples of model calculation and to assess errors with respect to the expected spacecraft altitude and off-nadir angle values of Cassini mission.

Relative errors are evaluated with respect to theoretical IR that is just the convolution of (3) with (8), i.e.,

$$IR^{\text{Theo}}(\tau) = \int_0^{+\infty} P_{\text{FS}}(\tau')P_{\text{HI}}(\tau-\tau')d\tau' \tag{30}$$

where the convolution integral is computed numerically.

For example, with reference to the main system parameter of Table I and by considering an altitude of the spacecraft of 5000 km and an off-nadir angle of 0.15°, Fig. 2 shows the corresponding normalized theoretical IR, evaluated for various surface height rms values ($\sigma_h = 10$ m, 50 m, and 100 m). With respect to this reference curve and by considering $\sigma_h = 10$ m as a reference value, Fig. 3 shows the relative errors, in percentage, for all models. These results prove that also with low off-nadir angle, the nadir model cannot be used, whereas the Prony models give negligible errors also with few terms ($N = 2$ or $N = 3$). More terms ($N = 4, N = 5$) do not add any improvement. As expected, also the asymptotic model gives very high errors.

The mean integral relative error (MIRE) has been evaluated and reported in Table III for all models and for $\sigma_h = 10$ m. This is the integral value of the relative errors of Fig. 3, averaged over the time interval where the theoretical normalized IR is significant ($> 1e^{-3}$). This parameter has been chosen as the full indicator of the goodness of the model, and it will be used in the following.

In order to examine the behavior of models as a function of spacecraft altitude in the nominal Cassini altimeter range, the MIRE has been evaluated for the same off-nadir angle and shown in Fig. 4. As expected, the values of Fig. 4 follow those of Table III, being high for the nadir and asymptotic models and very low for Prony's approximation, for which no improvements can be noted with more than three terms.

However, Fig. 4 is interesting for another reason, since it shows that the model's errors are almost independent of spacecraft altitude, at least in the Cassini altimeter operating range.

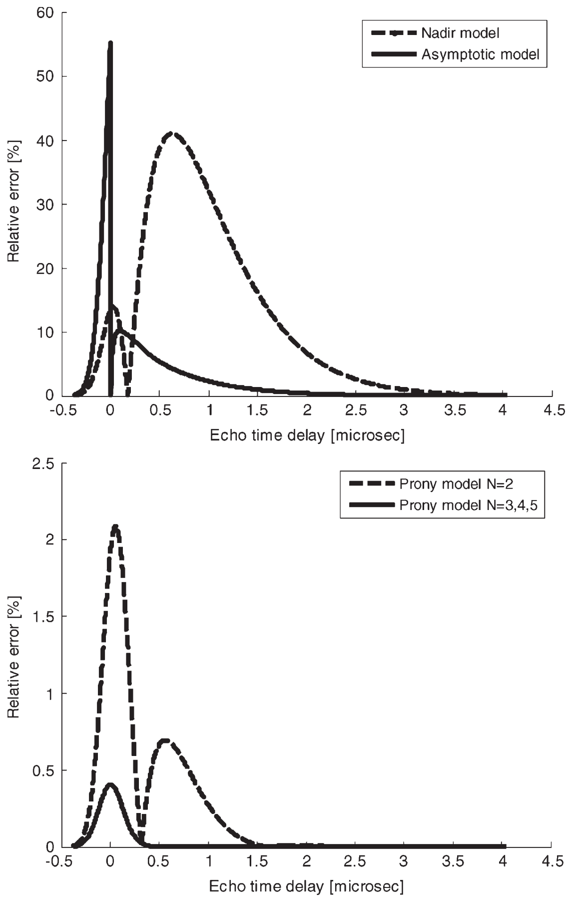

Fig. 3. Relative errors for all models ($H = 5000$ km and $\xi = 0.15°$).

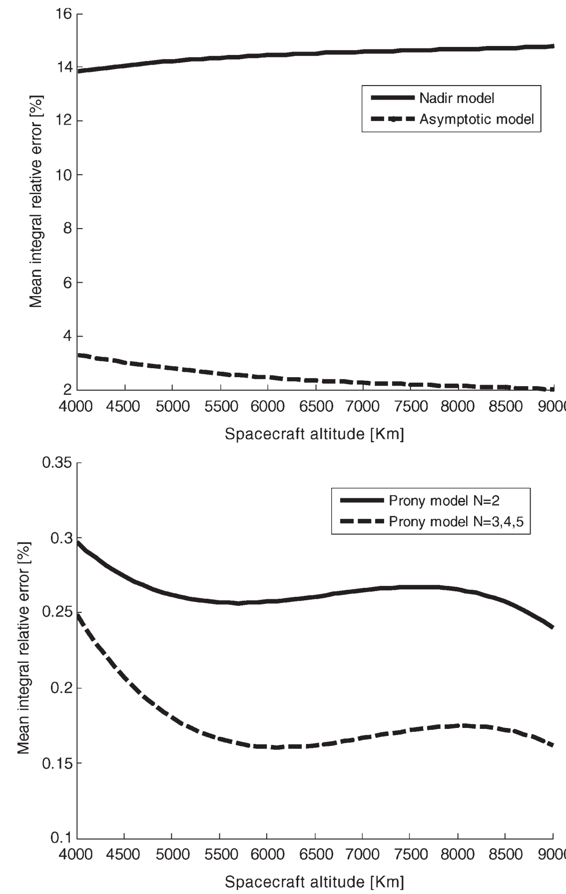

Fig. 4. MIRE for all models as a function of spacecraft altitude ($\xi = 0.15°$).

TABLE III
MIRE FOR ALL MODELS

Nadir	Prony				Asymptotic
	N=2	N=3	N=4	N=5	
11.471	0.113	0.027	0.026	0.026	2.829

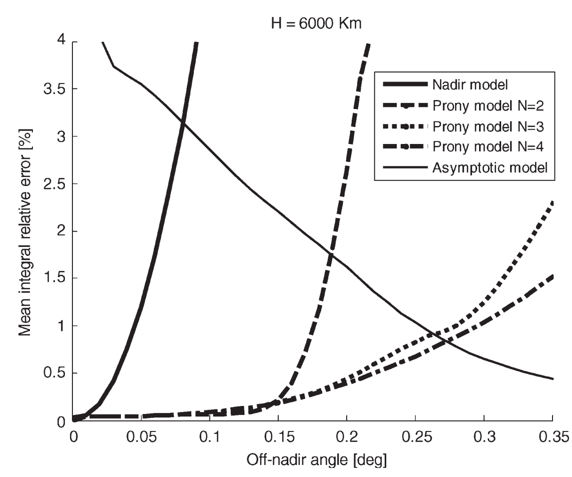

Fig. 5. MIRE for all models as a function of off-nadir angle ($H = 6000$ km).

The main dependence is instead on off-nadir angle, and it is summarized in Fig. 5, where MIRE is plotted for all models. Fig. 5 can also be used to fix threshold off-nadir values for switching among models, by using the crossing point between models and a criterion of a MIRE less than 1%. The evaluated threshold values and the corresponding model to be used are shown in Table IV.

V. HEIGHT RETRIEVAL ALGORITHM

In the previous paragraphs, an analytical approximated form of the averaged IR has been found, and its validity in the case of Cassini radar altimeter has been studied. The analytical expression depends on either system (such as off-nadir angle, transmitted bandwidth, antenna aperture and gain, spacecraft altitude, etc.) or terrain (such as mean and root mean squared height, sigma nought, etc.).

In the present paragraph, the "inverse" problem will be treated, which is the estimation of such parameters from real data acquired by the radar that are affected by thermal noise and, mainly, by speckle. This is a classical problem in the linear estimation theory, and several methods exist for inferring parameters of the underlying probability distribution from a given data set.

Among these, the MLE exhibits several characteristics which can be interpreted to mean that it is "asymptotically optimal" since it is asymptotically unbiased (its bias tends to zero as the number of samples increases to infinity) and it is asymptotically efficient, i.e., it achieves the Cramer–Rao lower bound when the number of samples tends to infinity [18]. This means that, asymptotically, no unbiased estimator has lower mean squared error than the MLE.

Given observations $(x_1, \ldots, x_N)$ depending on a set of parameters $(\theta_1, \ldots, \theta_M)$ and affected by noise with known

TABLE IV
THRESHOLD VALUES FOR OFF-NADIR ANGLE AND CORRESPONDING MODEL FOR ASSURING A MIRE <1%

Threshold off-nadir angle [deg]	Model to be used
$0 < \xi < 0.04$	Nadir
$0.04 \le \xi < 0.16$	Prony's N=2
$0.16 \le \xi < 0.26$	Prony's N=3
$0.26 \le \xi < 0.29$	Prony's N=4
$\xi \ge 0.29$	Asymptotic

probability density function, the MLE searches for the parameter values that maximize the likelihood function

$$L(\theta_1, \ldots, \theta_M) = f_\theta(x_1, \ldots, x_N/\theta_1, \ldots, \theta_M). \tag{31}$$

In our case, the compressed radar signal is digitized with a certain sampling frequency (f_s) and squared (power detected) so that each sample at a time $t_i = i/f_s$ is indicated with D_i.

Therefore, each sample D_i is exponentially distributed, with a mean equal to the ith sample of the averaged IR evaluated by using (13), (21), or (28) according to the off-nadir angle with the threshold values of Table IV.

In addition, for each burst, N_B pulses are available (typically 15 for the high-resolution altimeter mode) that can be exploited for making an incoherent summation before the height retrieval process starts. In this way, the ith averaged sample D_i can be approximately viewed as Gaussian distributed with

$$E[\overline{D}_i] = E[D_i] = IR_i \tag{32}$$

$$VAR[\overline{D}_i] = VAR[D_i]/N_B = IR_i^2/N_B. \tag{33}$$

By supposing the samples to be independent, the likelihood function becomes a product of N Gaussian probability density functions, such as

$$L(\theta_1, \ldots, \theta_M) = \prod_{i=1}^{N} \frac{1}{IR_i}\sqrt{\frac{N_B}{2\pi}} \exp\left[-\frac{N_B}{2IR_i^2}(\overline{D}_i - IR_i)^2\right]. \tag{34}$$

The maximization of such a likelihood function can be more easily performed by taking the derivation of the logarithm of the likelihood function itself

$$\frac{\partial}{\partial\theta_i}\ln(L) = 0 \tag{35}$$

where θ_i is the generic parameter to be estimated. The last expression can be rewritten as

$$\frac{\partial}{\partial\theta_i}\sum_{i=1}^{N}\left[\frac{1}{2}\ln\left(\frac{N_B}{2\pi}\right) - \ln(IR_i) - \frac{N_B}{2IR_i^2}(\overline{D}_i - IR_i)^2\right] = 0. \tag{36}$$

With simple calculations and by considering M parameters to be estimated, the last expression entails the solution of the following nonlinear system:

$$\begin{cases} \sum_{i=1}^{N} \frac{N_B\overline{D}_i^2 - N_B\overline{D}_i IR_i - IR_i^2}{IR_i^3}\frac{\partial IR_i}{\partial\theta_1} = 0 \\ \cdots\cdots\cdots\cdots \\ \sum_{i=1}^{N} \frac{N_B\overline{D}_i^2 - N_B\overline{D}_i IR_i - IR_i^2}{IR_i^3}\frac{\partial IR_i}{\partial\theta_M} = 0. \end{cases} \tag{37}$$

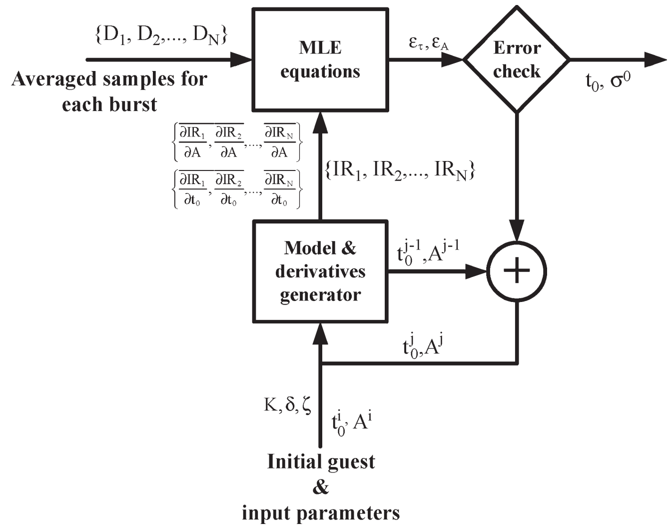

Fig. 6. Algorithm for actual implementation of MLE method.

The practical implementation of the MLE method differs from the last theoretical expression since its potential instabilities have to be managed. They are mainly due to the presence of amplitude terms in the expression of the derivatives of the model used that forces the use of simpler expressions for decoupling equations. To this end, the following suboptimal strategy has been followed.

1) Normalized derivatives have been used

$$\overline{\frac{\partial IR_i}{\partial\theta_M}} = \frac{\partial IR_i}{\partial\theta_M}\Bigg/\max\left(\frac{\partial IR_i}{\partial\theta_M}\right). \tag{38}$$

2) The IR_i^2 term has been neglected in the numerator since it is not multiplied by N_B.
3) The $\overline{D}_i$ term of the numerator has been simplified with IR_i in the denominator, since they are equal in the average.
4) Since low values of the remaining term IR_i^2 can cause instabilities in the estimate, the denominator has been substituted with a constant term, given by

$$C = \sum_{i=1}^{N} IR_i^2. \tag{39}$$

The final result for the actual implementation of the MLE method is therefore the following:

$$\begin{cases} \frac{1}{C}\sum_{i=1}^{N}(\overline{D}_i - IR_i)\overline{\frac{\partial IR_i}{\partial\theta_1}} = 0 \\ \cdots\cdots\cdots\cdots \\ \frac{1}{C}\sum_{i=1}^{N}(\overline{D}_i - IR_i)\overline{\frac{\partial IR_i}{\partial\theta_M}} = 0. \end{cases} \tag{40}$$

Of course, the last MLE equations are solved iteratively, following the scheme shown in Fig. 6, where only the echo time delay t_0 and the IR amplitude A are retrieved by using the MLE method.

By solving the equation set, at the nth iteration, two errors are evaluated (namely, $\varepsilon_\tau^{(n)}$ and $\varepsilon_A^{(n)}$), and their values are used for

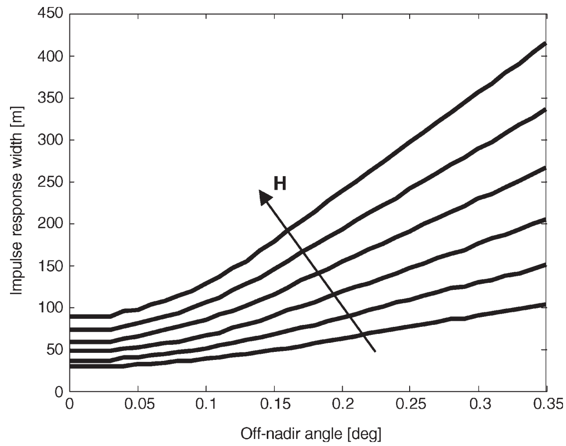

Fig. 7. IR time width (second central moment) as a function of off-nadir angle for various spacecraft altitude (H) values (4000 km up to 9000 km and step of 1000 km). The values have been evaluated on the basis of developed analytical models, and they are used for correcting the off-nadir angle to be used for the MLE procedure.

updating the actual estimates to be used in the next iteration, as

$$t_0^{(n+1)} = t_0^{(n)} + \varepsilon_\tau^{(n)}$$
$$A^{(n+1)} = A^{(n)} + \varepsilon_A^{(n)}. \quad (41)$$

The final values are reached when the two errors become lower than some fractions (0.01 for example) of signal sampling interval (200 ns for CASSINI) and signal maximum amplitude.

Some words should be spent to discuss on the influence of off-nadir angle on MLE performance. This angle was not included, at the moment, in the MLE estimation, but it is only used as a "perfect" parameter to generate IR and its derivatives. In other words, it was preferred to rely on a high degree of accuracy of spacecraft attitude control system instead of overloading the MLE procedure from a computational point of view, making the convergence more difficult. Nevertheless, there could be some other effects that would significantly affect the estimation performance such as the presence of local terrain slope that acts like an equivalent off-nadir angle. The final results on the received echo are a higher degree of echo time spread resulting in an increasing of received pulsewidth that, if not compensated, can degrade significantly the estimation performance. A method for managing this problem is correcting the off-nadir angle on the basis of the measured pulsewidth and following a relationship retrieved by using the analytical models. As a measure of received pulsewidth, the evaluation of signal second central moment can be used.

Fig. 7 shows the IR time width (second central moment) as a function of off-nadir angle for various spacecraft altitude values. The values have been evaluated on the basis of developed analytical models, and they can be used for "adjusting" the off-nadir angle to be used for the MLE procedure.

The actual performance of the implemented method can be evaluated via simulation, since the Cramer–Rao bound can give optimistic values in this case.

Single Cassini radar pulses can be generated by evaluating the theoretical IR through numerical computation of convolution integral. These values are used to generate exponentially

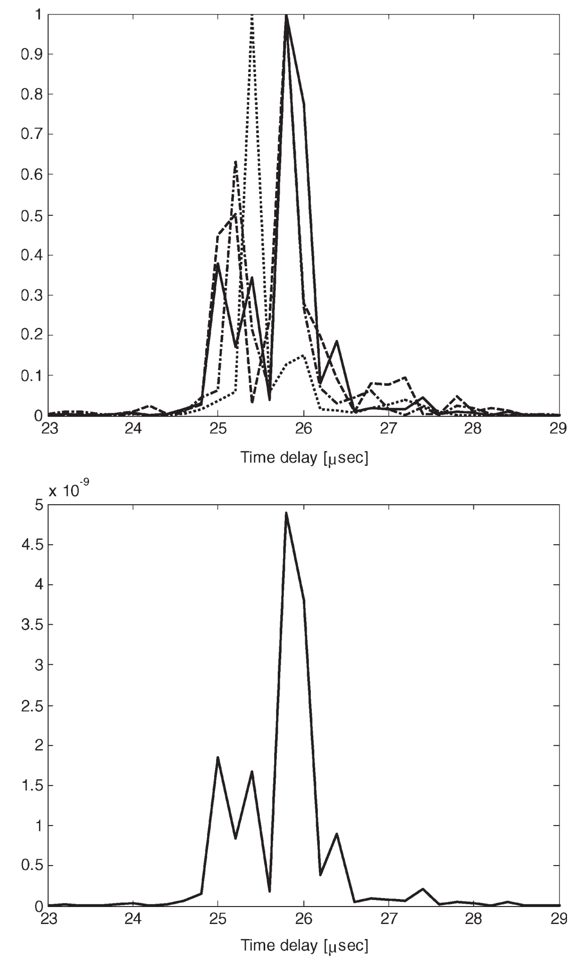

Fig. 8. Simulated Cassini radar echoes with $H = 5000$ km and $\xi = 0.15^\circ$—(top) some realizations and (bottom) the averaged echo.

distributed variables that represent a simulated radar echo. Following Cassini radar timing, fifteen echoes are therefore incoherently summed for simulating the averaged pulse obtained for each burst. An example of the obtained results is shown in Fig. 8.

The MLE algorithm has been applied to 1000 simulated bursts for each off-nadir angle, and altitude values in the operating range of Cassini radar and the obtained statistical results are shown in Figs. 9 and 10 in terms of mean and standard deviation of error height and normalized sigma nought.

A critical aspect of the proposed method is the choice of initial guess of the estimation cycle, particularly for low altitude and small off-nadir when the echo is expected to be very narrow.

To this aim, the followed strategy is based on integral measurements done on the received data corrected by means of analytical models.

In more detail, as far as the time delay is concerned (t_0^i of Fig. 6), for each averaged burst, the centroid values are evaluated, i.e., the samples that balance the energies on the right and left sides. These values are then decreased by a factor evaluated by means of developed analytical models and that accounts for the difference between the true time delay and the centroid. The delay correction is shown in Fig. 11 as a function of off-nadir angle and for various spacecraft altitude values.

The same procedure for evaluating the initial guess of echo amplitude, i.e., A^i of Fig. 6, can be followed. Since the MLE

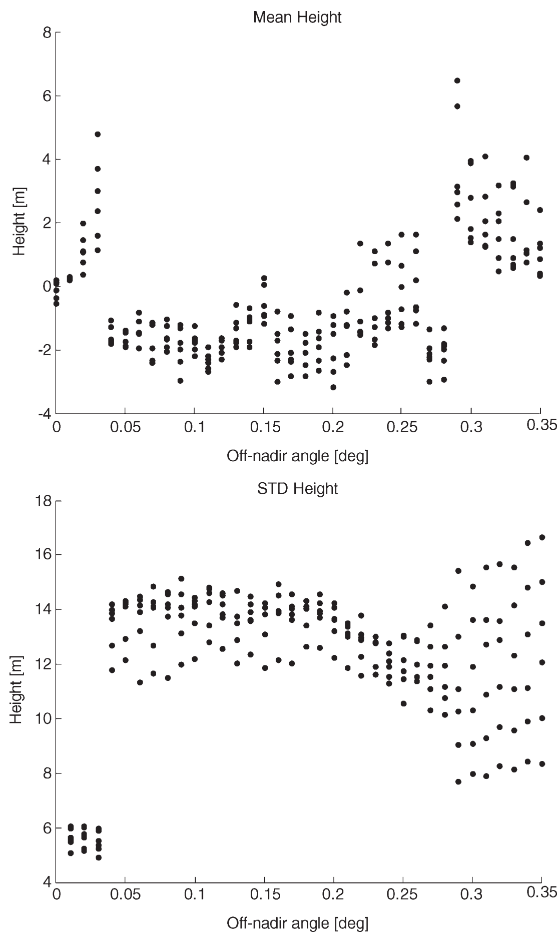

Fig. 9. Height error: Statistical results obtained by applying the MLE algorithm (1000 simulations) as a function of off-nadir angle for various spacecraft altitude values (4000 km up to 9000 km with 1000-km step)—(top) mean and (bottom) standard deviation values.

algorithm works with normalized waveforms, the initial amplitude value should be chosen in order to fix the resulting model amplitude equal to one. This can be done by using the developed analytical model, as shown in Fig. 12, where the initial values for getting a resulting model amplitude equal to one are plotted, as a function of off-nadir angle and for various spacecraft altitude values.

VI. Conclusion

An analytical model of the average return power waveform, valid for near-nadir altimetry measurements, has been developed in order to cope with the particular operating conditions of Cassini mission. The model is based on the same general assumptions of the classical Brown's model commonly used for oceanographic applications on Earth but exploits an approximation of the flat surface response by Prony's methods. The analytical model has been compared with numerically evaluated solutions, and it has been found that the MIRE can be kept below 1% by changing the Prony's approximation degree from 2 up to 4 to compensate the increasing off-nadir angle. This comparison has also been extended either to very low off-nadir angle values, where a closed form of the average return power already exists (nadir model), or to higher values where a different asymptotical approximation based on Laplace's method can be used.

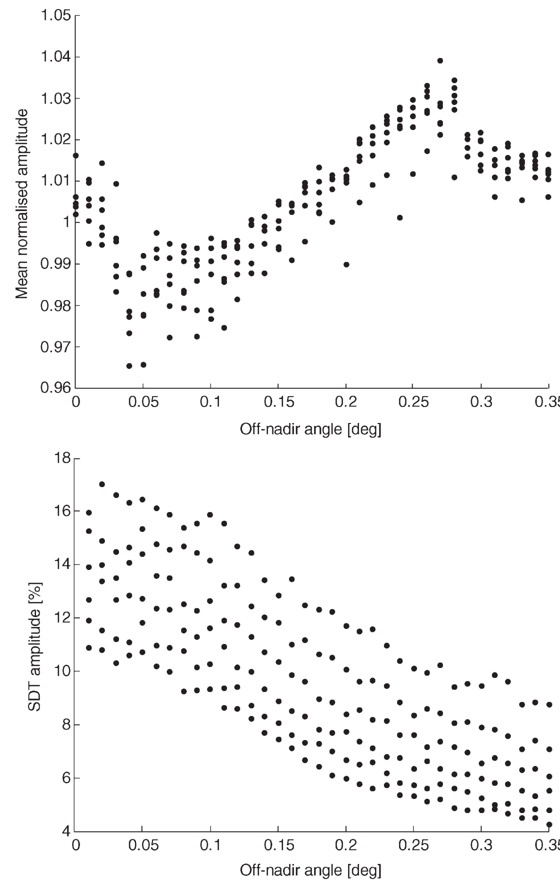

Fig. 10. Normalized sigma nought error: Statistical results obtained by applying the MLE algorithm (1000 simulations) as a function of off-nadir angle for various spacecraft altitude values (4000 km up to 9000 km and 1000-km step)—(top) mean and (bottom) standard deviation values.

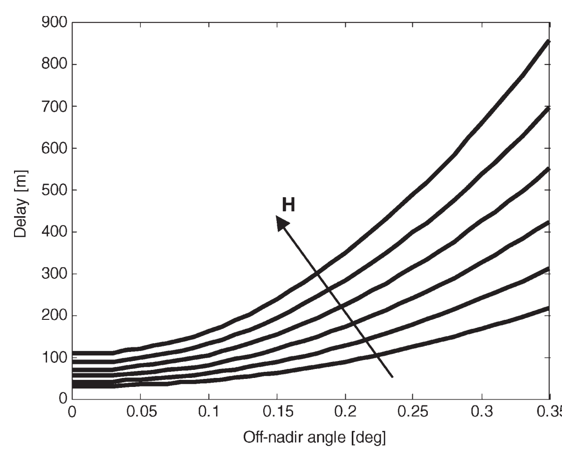

Fig. 11. Delay between waveform centroid and true height evaluated on the basis of developed analytical models, as a function of off-nadir angle and for various spacecraft altitude (H) values (4000 km up to 9000 km and step of 1000 km). The corresponding time delay is used for initializing MLE algorithm.

The error analysis allows switching among three different analytical models according to the current off-nadir angle of the measurements, as reported in Table IV. In addition, in order to infer the significant geophysical parameters describing the surface's topography from the altimetry data, an MLE method has been implemented. This algorithm will be used to process actual data of the Cassini mission and to produce standard

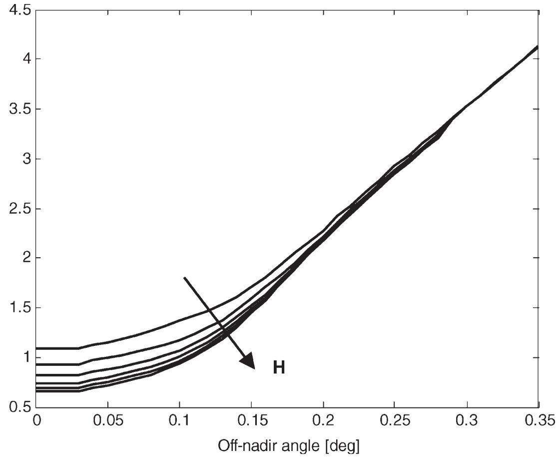

Fig. 12. Model amplitude values for getting a resulting normalized model, as a function of off-nadir angle and for various spacecraft altitude (H) values (4000 km up to 9000 km and step of 1000 km).

altimetric Cassini products (Altimeter Burst Data records) to be archived in Planetary Data System nodes. The performance of the proposed algorithm has been evaluated through simulation, and the results are shown in Figs. 9 and 10 for various off-nadir angle and altitude values in the operating range of Cassini radar.

As far as the retrieval of height is concerned, the mean value is between ± 6 m, almost independently of the used model, whereas the standard deviation is about 5 m for nadir model, 15 m for Prony's approximation model, and spreads from 10 m up to 25 m for asymptotic model depending on spacecraft altitude.

As far as the retrieval of sigma nought is concerned, the mean normalized values are between $\pm 4\%$, whereas the standard deviation shows a decreasing behavior for increasing off-nadir angle values starting from about 20% at nadir up to 4% at $0.35°$.

ACKNOWLEDGMENT

The authors would like to thank their colleagues involved in the Cassini mission for their fruitful collaboration and suggestions. During last year, the authors participated with the Cassini Altimetry Working Group, which has made great efforts for the assessment of processing algorithms for the retrieval of Titan's surface height. This team has been initiated by a joint agreement between ASI and NASA, and it is coordinated by E. Flamini and S. Wall. The authors would also like to thank all the people involved in this group, particularly R. Seu, R. Orosei, P. Callahan, H. Zebker, W. T. K. Johnson, and Y. Gim.

REFERENCES

[1] C. Elachi, M. D. Allison, L. Borgarelli, P. Encrenaz, E. Im, M. A. Janssen, W. T. K. Johnson, R. L. Kirk, R. D. Lorenz, J. I. Lunine, D. O. Muhleman, S. J. Ostro, G. Picardi, F. Posa, C. G. Rapley, L. E. Roth, R. Seu, L. A. Soderblom, S. Vetrella, S. D. Wall, C. A. Wood, and H. A. Zebker, "The Cassini radar Titan mapper," *Space Sci. Rev.*, vol. 115, no. 1–4, pp. 71–110(40), Nov. 2004, Springer.

[2] C. Elachi, E. Im, L. E. Roth, and C. L. Werner, "Cassini Titan radar mapper," *Proc. IEEE*, vol. 79, no. 6, pp. 867–880, Jun. 1991.

[3] G. Franceschetti, P. S. Callahan, A. Iodice, D. Riccio, and S. D. Wall, "Titan, fractals, and filtering of Cassini altimeter data," *IEEE Trans. Geosci. Remote Sens.*, vol. 44, no. 8, pp. 2055–2062, Aug. 2006.

[4] M. Abramowitz and I. A. Stegun, Eds., *Handbook of Mathematical Functions*. New York: Dover, 1972.

[5] G. Franceschetti, A. Iodice, M. Migliaccio, and D. Riccio, "Scattering from natural rough surfaces modeled by fractional Brownian motion two-dimensional processes," *IEEE Trans. Antennas Propag.*, vol. 47, no. 9, pp. 1405–1415, Sep. 1999.

[6] L. Borgarelli, G. Picardi, R. Seu, and E. Zampolini Faustini, "Altimetry in the Cassini mission," in *Proc. IGARSS*, Florence, Italy, 1995, pp. 1598–1600.

[7] G. S. Brown, "A useful approximation for the flat surface impulse response," *IEEE Trans. Antennas Propag.*, vol. 37, no. 6, pp. 764–767, Jun. 1989.

[8] G. S. Brown, "The average impulse response of a rough surface and its applications," *IEEE Trans. Antennas Propag.*, vol. AP-25, no. 1, pp. 67–74, Jan. 1977.

[9] A. Lee, *Cassini Orbiter Functional Requirements Book, Accuracy Requirements and System Capabilities, Rev. D*. Pasadena, CA: Jet Propulsion Lab., Nov. 7, 1997. CAS-3-170.

[10] C. H. Davis, "Satellite radar altimetry," *IEEE Trans. Microw. Theory Tech.*, vol. 40, no. 6, pp. 1070–1076, Jun. 1992.

[11] F. B. Hildebrand, *Introduction to Numerical Analysis*. New York: McGraw-Hill, 1956, pp. 378–382.

[12] J. I. Lunine and L. A. Soderblom, "Cassini-Huygens investigations of satellite surfaces and interiors," *Space Sci. Rev.*, vol. 104, no. 1–4, pp. 191–208, Jul. 2002.

[13] E. Montefredini, F. Morelli, G. Picardi, and R. Seu, "A non-coherent surface backscattering model for radar observation of planetary bodies and its application to Cassini radar altimeter," *Planet. Space Sci.*, vol. 43, no. 12, pp. 1567–1577, Dec. 1995.

[14] R. K. Moore and C. S. Williams, "Radar terrain return at near-vertical incidence," *Proc. IRE*, vol. 45, no. 2, pp. 228–238, Feb. 1957.

[15] D. O. Muhleman, A. W. Grossman, B. J. Butler, and M. A. Slade, "Radar reflectivity of Titan," *Science*, vol. 248, no. 4958, pp. 975–980, May 25, 1990.

[16] M. H. Newkirk and G. S. Brown, "Issues related to waveform computations for radar altimeter applications," *IEEE Trans. Antennas Propag.*, vol. 40, no. 12, pp. 1478–1488, Dec. 1992.

[17] C. M. Bender and S. A. Orzag, *Advanced Mathematical Methods for Scientists and Engineers*. New York: McGraw-Hill, 1978.

[18] A. Van Trees, *Detection Estimation and Modulation Theory (Part I)*. Hoboken, NJ: Wiley, 1968.

Giovanni Alberti received the degree (*cum laude*) in electronic engineering (telecommunication specialization) from the University "Federico II" of Naples, Naples, Italy, in 1989.

He is currently with the Consortium for Research on Advanced Remote Sensing Systems, Naples. He is Responsible of research projects sponsored by the Italian and European Space Agencies and NASA. In the last years, he was involved in both the Mars (MARSIS and SHARAD) and Saturn (CASSINI) missions. He has lectured on "Aerospace Remote Sensing Systems" at the Seconda Università di Napoli, Naples. His main research activities have dealt with the analysis and design of microwave sensors, such as synthetic aperture radar, radar altimeter, ground-penetrating radar, and image processing for use in the field of remote sensing. He is member of Cassini Radar and SHARAD Science Teams.

Luca Festa received the B.S. degree in aerospace engineering from the University of Naples "Federico II," Naples, Italy.

In 2002, he joined the Consortium for Research on Advanced Remote Sensing Systems, Naples. As a Researcher in the Microwave Division, he worked on radar systems performance evaluation and calibration and on data processing algorithms development. He is currently working with the European Space Agency, ESRIN, Frascati, Italy, as a System Engineer. His special research interest includes radar altimetry for planetary and Earth observation.

Claudio Papa received the Master's degree in aeronautics engineering from Federico II University, Naples, Italy, in 2002.

He was with the Second University of Naples, Naples, during 2002, working on voltage regulation for microsatellite power supply subsystems. By the end of 2002 up to now, he is with the Consortium for Research on Advanced Remote Sensing Systems, Naples. He is involved in radar data processing of major projects for planetary exploration, such as Marsis and Cassini altimeters. He has designed the processing algorithm for the Sharad level 1b processor.

Guido Vingione received the M.S. and Ph.D. degrees in aerospace science and technology from the Second University of Naples, Naples, Italy.

Since 2004, he has been with the Consortium for Research on Advanced Remote Sensing Systems, Naples, where he held the position of Research Scientist in the Radar Systems Division. His research focused on the development and testing of algorithms for radar altimeter systems waveforms modeling. He is currently working with the European Space Agency, where he holds the position of Ground Segment System Engineer. His research interests include signal processing, data exploitation, and waveform design in the context of radar system performance.