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Echoes Generation Systems: SHARAD Experience

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ABSTRACT

NASA's Mars Reconnaissance Orbiter mission, MRO, scheduled for launch in 2005, will be equipped with a sounder to find subsurface water and ice: Shallow Radar, SHARAD. This radar has been developed by Alenia Spazio and funded by Italian Space Agency, ASI.

An integral part of such kind missions is the development of an EGSE (Electrical Ground Support Equipment) capable to test all the radar functionalities.

CORISTA has been responsible to define the EGSE technical requirements and to design, build and test the Mars Echoes Generation System (MEGS).

This paper describes the activities developed and the results obtained during the test campaigns of SHARAD. An architectural description of the MEGS will be given with emphasis on the technical aspects related to the signal generation of Mars Echoes and possible operating modes.

Keywords: EGSE, Radar, Real Time Processing.

1. INTRODUCTION

All spaceborne radars need to be tested not only to verify that the specified performances are verified but even to test the postprocessing chain of the acquired data under all possible operating and environmental conditions. In order to perform such functional and performance tests of Sharad an EGSE has been set up. Two main subsystems constitute the EGSE: the spacecraft telecommand and telemetry simulator and the Mars Echoes Generator System, MEGS. In this work the main features and technical challenges that has brought to the development of MEGS are described.

CO.RI.S.T.A. was designed to be responsible for the MEGS in order to: define its main duties and the technical specifications; define the architecture; design and develop the system: both Hw and Sw.

MEGS has in charge the digital data handling and data conversion in interfacing SHARAD front end electronic.

Furthermore it had to be compliant to the following general requirements:

- support all the Operative Modes and OperativePhases of SHARAD;
- implement an entire operative orbit (30 min);
- work in open loop configuration using simulated transmitted pulse;
- be synchronized with the instrument using the PRI and Timing Signals from Sharad;
- be synchronized with Sharad also in open loop configuration;
- be able to acquire, store and graphic digital and analog signal (test point);
- be able to decode and follow the instructions contained in the sharad operative sequence table (OST);
- be able to modify the PRF during the entire operative orbit;
- be able to achieve a time stability lower than 1/(80 MHz)
- have a PRI counter.

The operative Modes supported by MEGS, and described later, are: acquisition of the pulses transmitted by Sharad, openloop, closedloop.

From Sharad point of view Megs represents MARS surface, in that it simulates the echoes that would be received by the radar when sharad's pulses are reflected back by a target of given characteristics. This simulation is achieved by generating a signal in a given time window and with appropriate power.

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Such echoes have to be generated accordingly to different operating conditions and injected into the receiver chain of Sharad to perform various tests. Such echoes generator system is by themself a radar that operates synchronously to the unit under test. In this paper a challenging experience of one of such system is reported.

MEGS generates echoes by the use of user defined scenario files generated by means of a Mars surface simulator developed in Matlab; it has also the possibility to simulate the additional delay due to ionosphere (included in the scenario files). Furthermore it has the possibility to open the echoes window in a settable position into the PRI according to the information set by simulator.

In the MEGS hardware architectural design described below, it has been considered the possibility to work in two main configurations: Open Loop and Closed Loop (Figure 1). In the closed loop configuration the signal is taken directly from SHARAD by means of a RF-FEE and, by means of the MEGS RF TX chain, its power is brought to an appropriate level at the input of an analog to digital converter, ADC. Once the signal has been digitized it is processed by the MEGS digital section where a FFT, IFFT and multiplication for the MARS transfer function are performed. A digital to analog converter transmit the processed signal back to Sharad. A galactic noise, generated either digitally or by means of an analog white noise source, is generated and added to the signal. These signals are combined by means of a RF RX chain before to be transmitted back to Sharad by means of the above mentioned RF-FEE. In Open Loop configuration only some of the steps performed in the closed loop configuration are performed: it is to insert a simulated TX pulse immediately after the ADC or to insert a simulated echoes before DAC (Figure 1).

Furthermore Megs must acquire and digitize TX signals using the 80 MHz timing signals from Sharad and it must be able to treat one transmitted pulses in each PRI. The digital noise consists of a vector of 94400 samples; as described in detail in the following paragraphs, these samples are computed in a way that it is completely transparent to the user and preloaded in the digital subsystem before that the sequence of tests to be performed starts. The loading of noise samples in the digital memory is organized as follows: the samples are subdivided in several different blocks in such a way that only the address of each block is necessary to furnish to the digital subsystem to make the corresponding samples to be generated. Such approach is particularly useful when the samples considered are relative to a realization of a white gaussian noise in order to increase the randomness of the noise generation among the different PRI. The noise injection starts some PRI before the Operational Modes described in the OST of Sharad in order to ensure system robustness. The timing and control unit shown in Figure 1 performs the following tasks: generates all timing and frequency signals for the MEGS starting from those provided by SHARAD timing interface; set the attenuation factors of the digital step attenuators of the RF-FEE chains; trigger the ADC and the DAC; switch from closed loop to open loop and vice versa during the set up.

Nowadays such kind of architecture is quite common as many electronic subsystems perform digital processing tasks by means of FPGA, but at the time the design here discussed was completed, end of 2002, it was still a technological innovation of great importance has pointed out in [1].

The paper is organized as follows: in section 2 and 3 the two main MEGS's subsystems, Radio Frequency Front End (RF-FEE) and digital subsystem, are described in detail; in section 4 and 5 the main characteristics of the man-machine software interface and those of Mars surface simulator are discussed; in section 6 the main results obtained during the Sharad test campaigns are discussed.

2. **RF-FEE** Architecture

The main purposes of the RF-FEE are: to adapt the power of the signal transmitted by Sharad to the maximum one allowed by the ADC, in order to optimize its dynamic range, and to generate echoes and noise signals in respect of the simulated SNR. To reach such goals this subsystem is composed by Rx and Tx chains built with programmable attenuators and switches controlled directly by the digital subsystem according to the directives furnished by the simulator.

The Tx chain is shown is Figure 2. An amplifier, a fixed and a variable attenuator adapt the level of the received signal to the input of the ADC. Besides a switchable antialiasing low pass filter, placed before the ADC, have been placed to filter out spurious signals coming from the radar.

The 27 dB gain of the amplifier A1 combined with the 5-35 dB of the attenuators FA1 and DSA allow to adapt the -20 dBm nominal input signal power to the -2 dBm maximum input power of the ADC with the capability to adapt signals whose power could rise up to 10dBm.



Figure 1: Megs general architecture.

Analog test points (TP1, TP2, TP3) have been placed along the chain to permit access to the RF signal in different points of the TX chain.



The RX chain (Figure 3) is composed by three channels summed by means of two combiners: two of them generate, respectively, the Mars echo (MOUT) and a digital noise (NOUT) and the third is an analog white noise source (ENR). In this way it is possible to generate noise or in digital or in analog, or even generate an interferer by means of an appropriate choice of the samples of the NOUT channel.

Each channel has a variable digital step attenuator that allows to set the right power level of the signal in respect to the others in order to reach the desired SNR at the Rx chain output.

Programmable switches, directly controlled by the digital subsystem, allow to select the desired signal path and to bypass the antialiasing filters placed at the output of the digital to analog converters, DAC.

Even in the Rx chain directional couplers allow to test the RF signals in different points of the chain: TP7, TP8, TP9, TP10.

The output level of the channels MOUT and NOUT, coming from DAC, is 4dBm and they can be attenuated separately.

The SNR attainable by the Rx chain is in the range -35dB to +35dB when the signal Nout is considered as noise source, and in the range -20 dB to +40 dB when the analog noise source ENR is enabled; an amplifier placed at end of the chain allows the power of the output signal to be hold between -60dBm to 0dBm by taking into account the insertion loss of the component of the chain.



Figure 3: Megs Rx chain.

Because the MEGS RF chains share the same radiofrequency path that constitutes the link between Megs and Sharad and in order to allow autocheck capabilities to the MEGS, the above mentioned RF chains are connected to Sharad by means of the network of switches shown in



Figure 4. The digital subsystem provides the synchronization of the control signals of the switches for a proper work of the branch.

Figure 4: Connection of MEGS RF chains to Sharad.

The test point TP0 allows to monitor the signals transmitted by Sharad.

3. Digital Subsystem Architecture

The digital subsystem of the MEGS consists of two digital boards both based on Virtex-II FPGA technology by Xilinx: GVA-350, V2MB-1000. An overview of the tasks that each board has to perform is shown in Figure 1. Basically the V2MB1000 performs the loading of data necessary to the digital subsystem to perform its funcionalities according to the modes described in the OST file. The main characteristics of the GVA-350 board are:

- Composed by 3 FPGA: 2 Virtex II (XC2V4000-4) and 1 Spartan II (XC2S100-6) called, rispectively, AC, Analog Control, DP, Data Processing, and EI, External Interface;
- Buses among the FPGA:
 - XEBUS: 43 bit bidirectional bus that connect all the three FPGA;
 - XBUS: 160 bit bidirectional bus between AC and DP;
 - FCBUS: 16 bit bidirectional bus between DP and EI;
- 4 ADC and 4 DAC connected to AC;
- ADC based on AD9432: 12 bit resolution, max sampl.freq. 105 MHz, Bw= 500 MHz, ENOB=11;
- DAC based on AD9752; 12 bit resolution, max sampl. Freq. 125 MHz, SFDR > 63dBc;
- 16 bit LVDS Rx and 16 bit LVDS Tx external channels;
- 48 bit bidirectional bus between EI and external connectors;

- Two 512kx18 SDRAM bank memory;
- USB interface.
- The main characteristics of the V2MB1000 board are:
- Composed by 1 Virtex II (XC2V1000-4);
- 2MBx16 DDR SRAM;
- 16 bit LVDS Tx and RX port with control;
- 106 bit bidirectional bus with external connectors;
- RS232 port, USB port, 10/100 Ethernet port.

In Figure 5 a detailed architecture of the digital subsystem is shown; in this figure the main memory buffers, digital data transfer channels and board level buses are shown. All the digital subsystems and funcionalities shown have been realized in VHDL.



Figure 5: Digital subsystem detailed architecture.

As shown in Figure 1 three digital test points have been implemented. Their purpose is to acquire data and send them to MEGS workstation, MWS, to be processed and showed on-line in time, frequency and time-frequency (off-line) domains. The funcionalities of each digital test point change according to the Megs configuration selected by the user:

- TP4: in closed loop it stores the data acquired by the ADC up to the entire operative orbit (30 min) by transmitting them to a disk array controller; the data are transferred to the MWS to allow on-line visualization. It is possible to enable this feature by software in real time. In open loop configuration the data of simulated echoes are read by a disk array and injected in the MEGS by means of this test point.
- TP5: both in closed loop and open loop configuration, this test point transmits the result of FFT to the MWS to allow its on-line visualization. On command, it stores in memory the results of up to 100 PRI (software selectable).
- TP6: in closed loop configuration it stores the data transmitted to the DAC up to the entire operative orbit (30 min) by transmitting them to a disk array controller; the data are transferred to the MWS to allow an on-line visualization of the signal to be generated. It is possible to enable this feature. In open loop configuration the data of simulated echoes and noise to be generated are read by a disk array and injected in the MEGS by means of this test point.

A player on MWS allows to play off-line all data stored in the digital test points.

The implementation requirements upon which the digital subsystem has been designed are the following: the length of the pulse transmitted by Sharad is 85 us; by taking into account the sampling frequency of 80 MHz this results in 6800 samples to be acquired. Because the convolution with the Mars transfer function is performed in the frequency domain the next power of 2 has been considered: 8192. On these samples an FFT is performed by considering the hermitianity property of real signal. The scenario files are composed by 8193 complex samples disposed in such a way to take advantage of the above mentioned algorithm; the accuracy of the positioning of the scenario is 1/(80 MHz). This scenario is multiplied for the 8192 complex points output of the FFT. The scenarios are organized in files containing records relevant to each PRI. The possibility to use a given set of scenario files (e.g.: 100 PRI) during all the simulation time (e.g. 30 min.) has been implemented. An IFFT on 8192 complex samples is performed on the result of the above mentioned multiplication and even in this case the algorithm has been optimized for real signal using the hermitianity property. All the steps that has brought to the mentioned optimization of the FFT – multiplication in frequency domain – IFFT are omitted for brevity; the basic result to point out here is that it is possible to pipeline the entire algorithm with a 5 stage pipeline[2-3]. Both the FFT and IFFT are performed in Hw by means of an intellectual property core by Xilinx based on a pipelined, decimation in frequency, radix-2 FFT algorithm. It is important to note that there is an initial latency, that must be determined, before the first sample of the FFT of the first block of data is available at the output of this IP core. To this end simulations have been performed to determine the exact number of clock cycles of this initial latency. Even if the value for many values of N, points on which compute the FFT, have been determined, here are reported only the value for N=8192: 18533 clock cycles.

The timing of the tasks that MEGS must perform in each PRI in worst case condition are described below. These conditions have been identified with the fastest PRF with which Sharad, and MEGS as well, must be able to work: 775.19 Hz. This PRF is the worst case because the time available to complete and synchronize the great number of tasks that must be executed, is the shortest. The time sequence of such tasks with an estimation of the time spent to accomplish them and with an indication of the digital resources (Figure 5) employed are listed below:

- Input Signal Acquisition: starting from an external trigger signal, coming from the timing and control unit, the pulse generated by SHARAD is acquired and transferred at the input of the FFT module, stored in the buffer memory MEM-DP (Figure 5), for the on-line visualization on MWS if enabled, and saved on the digital test point TP4, for post processing, by means of CH-TP6 (Figure 5). The acquisition of 8192 real samples @12 bit @ 80 MHz is 102.4 us (105 us) long;
- MARS Transfer Function: the same trigger that starts the acquisition of the Sharad transmitted echo, will start the transfer of the Mars transfer function samples from the disk array, with a minimum delay. The resources used in the transfer are: LVDS-RX and CHB (Figure 5). The samples to be transferred in each PRI are 8193 complex values @ 15 bit @ 50 MHz = 8193/50 = 163.86 us (166us);
- Digital Noise Generation: the noise samples are transferred, by means of CHA (Figure 5), from the BUFFER NOISE to the DAC-NOISE to be generated; at the same time these samples are stored on MEM-DP, for on-line visualization, and on TP6 digital test point by means of CH-TP6. Because the different realizations of the digital noise are generated by reading the same samples, divided in 256 different blocks, in an order defined by 256 different 8 bit indexes, what must be saved, both on TP6 and on MEM-DP for on-line visualization, are these 256 indexes only;
- Mars Echo Generation: the samples of the echo to be generated in the current PRI, stored in the previous PRI on BUFFER2-AC (Figure 5), are read and transferred to DAC-ECHO;
- End of FFT: at the end of the computation of the FFT, the 8192 complex samples are stored in BUFFER1-AC: 8192 Complex Samples @ 18 bit @ 50 MHz = 8192/50 = 163.84 us;

- FFT Post Processing: it will be executed in pipeline, the output will be a complex sample at each clock @ 50 MHz. The data processing requires to read the FFT results, Mars Transfer Function, sine and cosine values previously saved, respectively, in BUFER1-AC, BUFFER-MARS and MEM-AC. Time spent to complete the post processing is 8192 Complex Samples @ 16 bit @ 50 MHz = 8192/50 = 163.84 us. In this time estimation the 5 clock cycles spent to execute the frequency domain multiplication pipeline previously described have been discarded because of low order of magnitude in respect to the time spent to complete this task;
- IFFT Result: at the end of the IFFT, the samples are saved both on BUFFER2-AC and on BUFFER1-AC(Figure 5);
- Next PRI Echo samples: during the saving of next PRI Echo samples on Buffer1-AC, after that a given amount of them have been already saved, these samples are transferred and saved on DP FPGA by means of CHB. This task is possible because Buffer1-AC is a Dual Port buffer. At the same time next echo samples are saved both on MEM-DP for on-line visualization, if enabled, and on digital test point TP6 by means of CH-TP6. The samples to be transferred are 16384: 4 samples/clock @ 25 MHz = 163.84 us;
- TP5 Data Storing: The results of the FFT post-processing are transferred, by means of CHB and CH-TP6, respectively on MEM-DP for on-line visualization, if enabled, and on disk array for TP5, if enabled by the operator. The time estimates of the tasks described have been succesfully verified during Megs' test campaigns. From the description of these tasks it can be pointed out that the data transfer from the two disk arrays used in Megs have to be able to substain a high transfer rate. The disk array controller employed is a PCI board with standard ATA-133 hard disks interfaces for the array and FPDP interface for the real time data streaming. The order of magnitude of such data rate of both disk array controllers are, respectively: 50 Mbytes/s to save the digital test points data and 25 Mbytes/s to read ancillary data necessary to perform the convolution in the frequency domain.

4. Man Machine Interface

The software that constitutes the interface between the user and the MEGS has in charge to perform the following tasks:

- MEGS configuration management;
- Data acquisition and archiving;
- MEGS event logging and monitoring;
- Graphic acquired data in time domain, in frequency domain and in time vs frequency domain;
- Furthermore the software ensures the possibility to monitor all phases of the test campaign.

The configuration management is performed by means of ini files that allow the user to recall a previously saved Megs configuration or to generate new ones. This setting parameters facility ensures the possibility:

- to set the level of the digital step attenuator of the RF Tx chain (Figure 2), in order to optimize the ADC dynamic range;
- to set off-line as well as on-line almost all the elements of the switch net in real time;
- to set the power of the echoes generated by MEGS; this goal is reached by setting the right Digital step attenuators levels starting from Mars scenario informations (Figure 3). This settings are completely user transparent because they are managed directly by MMI.

As previously described, MEGS is able to operate in different configuration modes according to the test that need to be performed and described in the OST file. For each of them different settings of the MEGS' hardware are necessary in order to make it work properly. These settings are performed automatically by the MMI and only the input parameters applicable to the chosen configuration are capable to be modified by the user.

In order to work in accordance with the radar, MEGS needs to know in advance which are the operating modes that will be used by Sharad and as previously stated this function is accomplished by means of the OST file. Because this file describes the main characteristics of each operating mode to be used, it is necessary not only to load it into the MMI but even to load the scenario files, previously generated by means of the Mars simulator, that match those characteristics. The OST files is organized in rows and the MMI allows to load a scenario files for each row and perform an automatic check to ensures that the OST and scenarios loaded are compatibile on a row by row basis.

Furthermore during the setup phase the user may select if the noise source to be used in the RF Rx chain is the digital one and in such case select how to generate the noise samples: gaussian white noise, preloaded waveforms or load a user defined file with the desired samples to be generated.

The data coming from the digital test point TP4, TP5 and TP6 shall be acquired and showed in a graphical way in time domain, frequency domain and time vs frequency domain. Some of these plots are showed on-line while during off-line data processing a player is provided to allow the user to analyze all the PRI simulated by MEGS.

Another important feature that MEGS has is the ability to perform an automatic autocheck feature. As shown in Figure 2 and Figure 3 the RF-FEE subsystem comprises a great number of switches and digitally controlled attenuators; in order to check that each one of them is working properly, to verify the calibration of each RF chain and signal sources, and to avoid the possibility of faults before to execute critical test campaigns, this feature can be executed and analyze with the aid of graphics the results obtained in real time. The autocheck consists of 13 different tests to be performed together or separately according to the fault suspect conditions.

5. Mars Simulator

We introduce a SAR raw signal simulator, which computes the reflectivity map relying on an electromagnetic polarimetric model of an extended three-dimensional scene. It accurately evaluates the raw signal by means of the appropriate system function, analytically computed in the (two-dimensional) range-azimuth time domain, including the aberrations of the SAR system

We present now the general structure of our simulator. The height profile of the scene is described by an appropriate function, which may be prescribed either analytically or numerically. This profile is then approximated by square plane facets, large in terms of the incident wavelength but small when compared to the resolution length. Each facet is characterized by the coordinates of three vertices (or, alternatively, by the coordinates of its center and the associate normal) and by the electromagnetic parameters (permittivity e and conductivity) of the underlying material.

The SAR raw signal is the appropriate superposition of returns from each facet. Accordingly, a first problem is the computation of individual facet back-scattering, taking into account local angle of incidence, polarization of the incident wave, the facet's roughness, and any shadowing effect, if present.

Once the elementary fields back-scattered by each facet have been computed, their superposition is in order. Irregularities of the (macroscopic) terrain profile imply statistical displacements of the facet's vertices, which can be modeled by associating with each facet a random displacement of three of its four vertices. As an alternative possibility, we note that this vertex displacement is equivalent to moving the facet center and to changing the orientation of its normal. Accordingly, we can add a random displacement with an appropriate probability distribution function (PDF) to each facet position, as well as a random normal orientation.

The efficient summation of all returns is considered in the time domain. Range migration and curvature effects are included in the transfer function. Change of focus depth with the distance is also incorporated by a grid deformation in the transformed space.

The simulator involve numerous parameters that are necessary for the data result. This parameters are either the satellites parameters, the scenario parameters and the sensor parameters (Figure 6).

Figure 7 shows the block diagram of the simulator. The SHARAD scenario simulator is a file generator unit that taking in account of the relative position between the target and the sensor and the terrain back-scattered coefficient. The orbital parameter block evaluated the reference frame considering the satellites orbital parameters.

The Electromagnetic Parameter of Scenario Typology block taking in account the characterisation of the Martian surface composition: a basaltic composition in the southern hemisphere, and an andesitic composition in the north.

Although a multitude of different materials is present at the surface of Mars, few representative scenarios have been selected with different dielectric constants as most meaningful for electromagnetic studies. Table 1 shows a reference summary of the representative scenarios of the materials.

Ĺ		[A							
SATELLITES PARAMETERS				SENSOR PARAMETERS					
3397	Planet Radious [Km]		10	PRF [Hz]					
320	Orbital Minimum Distance [Km]		85	Pulse Length [micros]					
320	Orbital Maximum Distance [Km]		102.4	Window Length [micros]					
92.8	Orbital Inclination Angle [*]		15	Frequency Start [MHz]					
0	Antenna Elevation angle [*]		25	Frequency Stop [MHz]					
SCEN/	ARIOUS GEOMETRY		80	Frequency [MHz]					
0	Start Anomaly for Data Acquisition ["]		0	Integration Time [sec]					
2	Range Strip Length of Scenario [Km]		MARS	IONOSPHERE PARAMETERS					
2	Azimut Strip Length of Scenario [Km]		80	Equivalent lonosphere Layer [Km]					
100	Scenario Range Resolution [m]		3	Plasma Frequency [MHz]					
100	Scenario Azimuth Resolution [m]								
1	Azimuth Facet per Azimut Cell								
1	Range Facet per Range Cell								
	SCENARIOUS TIPOLOGY								

Figure 6: Input parameters interface of Mars simulator.



Figure 7: Mars simulator block diagram.

Category		3	Tan δ	Category		3	Tan δ
Andosita	I-1	5	0.004	Different	II-2	5	0.03
Andesne	I-2	8	0.004	Terrestrial	II-3	9	0.03
Basalts	III	7.1	0.014				

Table 1 - Reference categories of Martian crust materials

The Scenario Profile Block is introduced for describe the height profile of the scene by an appropriate function, which may be prescribed either analytically or numerically.

The Main Program utilize a block which evaluated the reflected field utilizing the facets method and the obtained data are utilized for generate the output raw signal file

6. Results

In this paragraphs the main and most meaningful results are shown; they have been chosen because best represent the performances achievable by MEGS. The graphs shown have been obtained by processing tha data acquired by Sharad and downloaded on a off-line post-processing workstation; the compression algorithm used is a standard one and written in Matlab.



Figure 8: Compressed image in closed loop (a superficial interface with a sub-superficial interface).



Figure 9: Compressed image in closed loop (4 point-targets).



Figure 10: Compressed received signal: more than 60 dB of dynamics after calibration.



Figure 11: Megs rack during test campaigns.

In Figure 11 the MEGS rack in its final configuration is shown.

7. References

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